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# Some Distributions of Postpartum Amenorrhea

**Abstract**

Two probability distributions to describe the mechanism of variation in the length of post-partum amenorrhea period have been proposed. The first model incorporates the assumption that the intensity of resuming menstruation after a live birth is a linear function of time elapsed from the date of birth. The second model is obtained by considering the variations in the risk of resuming menstruation, among women. Both the models have been applied to an observed set of data on the basis of frequency estimates of the parameters, for the purpose of illustration.

## Introduction

THE postpartum amenorrhea (P.P.A.) is the period of temporary sterility, immediately following the termination of a pregnancy during which the possibility of conception taking place, does not exist. This amenorrhea period is highly correlated with the time and extent of breast-feeding practices prevailing in different societies and is an important determinant of fertility. There are evidences of variations in the time and extent of breast-feeding practices within and across the countries, depending upon different socio-economic conditions. Generally, amenorrhea period varies from one month to two years (Bongaarts and Potter 1983).

The time of first ovulation after the birth of a baby is an important factor in fertility studies. Generally, it takes place within a few weeks before or after the first post-partum menses (Perez, *et al.* 1971). Hence the mean duration of post-partum amenorrhea is taken as a good indicator of the mean duration of the onset of ovulation. The sucking stimulus of breast feeding delays the return of menstruation and ovulation by raising the level of hormone, prolactin. Thus, the major determinant of prolonged anovulation is lactation, (Billewicz 1979; Habicht, *et al.* 1985; McNeilly 1977; Tietze 1961; Lesthaeghe and Page 1980; Jain, *et al.* 1970).

It is important in demographic research on fertility to devise a suitable model which takes into account the heterogeneity in time and pattern of infant feeding and also in the menstrual response to breast-feeding.

The wide variations in the duration *of lactation amenorrhea* are strongly correlated with the breast-feeding practices (Van Ginneken 1974, 1978). It has been commonly observed that women belonging to the family of good income level tend to supplement their children's

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diet much earlier, thus reducing the frequency and contents of breast-feeding, whereas the women from the poor family have to undergo prolonged and intense suckling by the hungry infants (Salber, *et al.* 1966; Tyson and Perez 1978; Ford and Kirn 1987). For an individual woman, the amenorrhea is affected not only by the duration of breast-feeding but also by the type and intensity of breast-feeding. Populations with the longer durations of breast-feeding experience prolonged amenorrhea and show positive correlation between lactation and amenorrhea, (Huffman, *et al.* 1987; Malkani and Mirchandani 1960; Perez, *et al.* 1971; Srinivasan, *et al.* 1989). However, the nutritional supplement programme may reduce breast-feeding (Popkin, *et al.* 1979).

In the past, several studies have been carried out to describe the distribution of post partum amenorrhea. Srinivasan (1966) assumed that amenorrhea period follows a discrete triangular distribution. Talwar (1965) and Yadava (1966) assumed this period to have assymmetric triangular and chi-square distribution respectively. Saxena and Pathak (1977) fitted a mixture of the two truncated Chi-square distributions. Billewicz (1979) suggested a model depending upon the proportions of feeding mothers. Barret (1969) studied the distribution of post partum amenorrhea as a modified Pascal distribution. Ginsberg (1972, 1973) suggested a general stochastic model using the data available on the length of partial and complete breast-feedings. However, in practice, the determination of partial and complete breast-feedings remain highly subjective and depends largely on age and social, economic, cultural and educational backgrounds of the respondent. Ford and Kirn (1987) proposed a mixture of type I extreme value distribution, also sometimes called Gumbel distribution (Johnson and Kotz 1970) to describe the distribution of post partum amenorrhea period. Pathak and Pandey (1984) fitted the mixture of the two displaced geometric distributions in order to estimate the proportions of the women with different breast-feeding patterns and intensity on a group of women to revive menses after the live birth; by using the simple information on the duration of post partum amenorrhea.

It may, however, be mentioned that none of the attempts were made to study the distribution of the post partum amenorrhea by taking into account the different heterogeneity factors governing the risk of post partum menses following the birth of a baby, using the simple information on the duration of amenorrhea period.

The aim of the present paper is to develop probability models of the amenorrhea period. While the first model aims to consider the intensity of resuming menses after a live birth to a woman as time dependent, the second model assumes that the intensity parameter varies among the women following a type-Ill gamma distribution.

The proposed distributions not only enable us to get the estimates of the proportions of the women with different breast-feeding patterns but also predict the behaviour of the risk of intensity to menses. Both the models, incorporate the experiences of obstetrics that there will be a minimum post partum amenorrhea period associated with each live birth irrespective of the breast-feeding patterns among the women. The theoretical frequencies are compared with the observed frequencies and the estimates of the parameters are obtained by using the method of minimum % (chi-square) procedure.

## 2. Distribution of Postpartum Amenorrhea

*Model I*

This model has been derived under the following assumptions on the lines of Pathak and Pandey (1984):

1. The mother breastfeeds the child atleast for $k$ months, in case it survives.
2. The minimum length of amenorrhea for the women after live birth is v months after which she is exposed to the risk of menstruation, where $v \leq k$.
3. After $k$ months the probability that the women breastfeeds as usual is $(1 - \alpha)$ and the probability that she decreases the intensity of breast-feeding by supplementing some outside food or milk is $\alpha$.
4. The risk of exposure to menses for the amenorrhic women with normal breast-feeding remains constant, say $\lambda$ in an infinitesimal unit of time, until she supplements the breast-feeding by some out side food.
5. In case the amenorrhic woman supplements breast-feeding with outside food, the risk of resuming menstruation, increases to $\lambda_1$ where $\lambda = C\lambda_1$, $0 < c < 1$.
6. $\lambda$ or $\lambda_1$-depends on time elapsed from the birth.

In case of the infant death, the breast-feeding adruptly stops and the menses may be resumed faster if it has not occurred earlier. Such cases are dealt with by increasing Xi and a to some extent. We assume that the infant mortality does not affect the distribution of amenorrhea period. It is, however, possible to account for the effect of infant deaths but the expression will be more complicated.

Let the random variable $x$ denote the length of post partum amenorrhea period following a live birth of order $i$ $(i \geq 1)$. Then, under the above assumptions, the probability function of $x$ is given by

$$P[x \leq x \leq x + dx] = \begin{cases} \lambda(x)\, e^{-\int_v^x \lambda(t)\,dt}, & v \leq x < k, \lambda > 0 \\ e^{-\int_v^k \lambda(t)\,dt}\left[\alpha\, \lambda_1(x)\, e^{-\int_k^x \lambda_1(t)\,dt} + (1-\alpha)\, \lambda(x) e^{-\int_k^x \lambda(t)dt}\right], & x \geq k\ \lambda_1 > 0, 0 < \alpha < 1, \end{cases} \tag{2.1}$$

where $\lambda(t) = a + bt$

and $\lambda(t) = c\lambda_1(t)$ where $c < 1$.

$$P[x \leq x \leq x + dx] = \begin{cases} (a+bx)\exp\left[-\{a(x-v) + \frac{1}{2}b(x^2 - v^2)\}\right], & v = x < k, \lambda > 0 \\ \exp\left[-\{a(k-v) + \frac{1}{2}b(k^2 - v^2)\}\right].\ \left[\alpha\left(\frac{a+bx}{c}\right)\right. \\ \exp\left\{-\frac{1}{c}\left(a(x-k) + \frac{1}{2}b(x^2 - k^2)\right)\right\} + (1-\alpha)(a+bx) \\ \left.\exp\left\{-\left(a(x-k) + \frac{1}{2}b(x^2-k^2)\right)\right\}\right], & x \geq k, \lambda_1 > 0, 0 < \alpha < 1, \end{cases} \tag{2.2}$$

where *a, b,* and c are constants, and $\lambda(t)$ and $\lambda_1(t)0$ denote the risk of exposure to menses at time *t* from birth for the women belonging to two patterns of breast feeding after a live birth.

For $\alpha$ - 0, the model reduces to a simple exponential distribution and for $\alpha = 1$ it reduces to a displaced exponential. For the present analysis, we have taken the group of women as homogenous with respect to their risk of exposure to the resumption of menses after live birth.

*Model II*

In model I, the risk of exposure to the resumption of menses is treated homogenous; but in practice, $\lambda$ and $\lambda_1$ vary among women. For such cases, we assume that $\lambda$, follows type-Ill distribution as below:

$$f(\lambda) = \frac{\lambda^{a-1} h^a e^{-h\lambda}}{\Gamma a} \tag{2.3}$$

and $a$ and $h$ are positive integer.

We further asume that $\lambda$ or $\lambda_1$ does not vary over time for the same woman. Thus, our model becomes:

$$p\,[x \leq x \leq x + dx] = \int_0^\infty f(x, \lambda)\, dF(\lambda)\,,\ \lambda > c$$

where

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda(k-v)} & \text{for } v \leq x \leq k,\ c \leq \lambda_1 \leq 1 \\ e^{-\lambda(k-v)}\,[\alpha\lambda_1 e^{-\lambda_1(x-k)} + (1-\alpha)\,\lambda e^{-\lambda(x-k)}]\,, & x \geq k, \lambda_1 > c,\ c < \alpha < 1 \end{cases} \tag{2.4}$$

$$p\,[x \leq x \leq x + dx] = \begin{cases} \dfrac{ah^a}{(h+x-v)}a+1,\ v \leq x < k \\ \alpha \begin{pmatrix} a \\ c \end{pmatrix} \dfrac{h^a}{\left(h + \frac{x-v}{c}\right)^{a+1}} = (1-\alpha)\ \dfrac{ah^a}{(h+x-v)^{a+1}},\ x \geq h, h > c \end{cases} \tag{2.5}$$

**3. Estimation of the Parameters and Illustrative Application**

*A. Parameters of Model I*

There are six parameters namely, v, *k*, $\alpha$, a, *b*, *c* in the distribution given by the expression (2.1). It may be noted that the maximum likelihood equation becomes quite intractable for estimating these parameters.

By assuming the empirical values of v and *k*, the frequency estimates of the parameters $\alpha$ and $\lambda_1$ can be easily obtained after getting the estimates of a, *b*, and *c* with the help of first three cell frequencies of the observed distribution. The first cell frequency , generally, implies the proportion of mothers resuming mentruation within the minimum period of lactation, without food supplementation. Here our purpose is to provide a simplest procedure of estimation so that the estimates so obtained can easily be used as pilot points in obtaining more efficient estimates.

It is assumed that v = 1, in the present case, because in normal situations, no woman will resume menses before one month. The value of $k$ depends upon the lactation practices in different societies. It has been observed that mothers in India, normally breastfeed at least upto three months till the child becomes able to take some soild food. We assume, therefore, $k$ = 3 months, for our purpose.

The proposed model is applied to the data of Saxena (1966) to see its validity in describing the observed distribution. In this survey, information pertaining to post-partum amenorrhea period was available for 369 women who had given atleast one live birth and were in the age group of 35 years at the time of inquiry. From the observed distribution of post-partum amenorrhea, reported in Table 1, we get the four relative frequencies as $f_1$ = 0.26558, $f_2$ = 0.24390, $f_3$ = 0.16260, $f_4$ = 0.10298. Now, equating these four observed relative frequencies with the theoretical ones of model (2.1) and solve them, we get the frequency estimates of the parameters as $\bar{a}$ = 0.1546, $\bar{b}$ = -0.00013, $\bar{c}$ = -0.6870, $\bar{\alpha}$ = 0.7042.

TABLE 1 : OBSERVED AND EXPECTED DISTRIBUTION OF POSTPARTUM AMENORRHEA FOLLOWING FIRST LIVE BIRTH

| *Duration of PPA (in months)* | *Number of Mothers* | |
|---|---|---|
| | *Observed* | *Expected* <br> *v = 1 month* <br> *k = 3 months* <br> *a = .1546* <br> *b = -.00013* <br> *c = .6* <br> *α = .70452* <br> (Based on frequency estimates) |
| 1-3 | 98 | 98.00 |
| 3-5 | 90 | 90.00 |
| 5-7 | 60 | 59.97 |
| 7-9 | 38 | 38.02 |
| 9-11 | 27 | 26.63 |
| 11-13 | 25 | 18.20 |
| 13-15 | 14 | 11.87 |
| 15-17 | 8 | 8.03 |
| 17-19 | 5 | 5.46 |
| 19-21 | 2 } 4 | 3.74 } 12.82 |
| 22 and over | 2 | 9.08 |
| Total | 369 | 369.00 |
| $\chi 2$ | 9.03 | |
| d.f. | 5 | |

The small value of $b$ suggests that the risk of exposure to menses may be treated as constant and equal to $\lambda$ = . 1550 and $\lambda_1$ = .2250 (approx.); for the present data $b$ is small and negative which indicates the fact that during the amenorrhea period, the risk of menses decreases and then stablises after some period.

The fit, on the basis of frequency estimates so obtained implies that the population of mothers is composed of two sub- groups comparising about 70% and about 30% of the population of mothers.

*B. Parameters of Model II*

The distribution given by the expression (2.5) involves 6 parameters, namely $\alpha$, *a, b, c,* v, and *k*. Here also, we assume the empirical values of *v* and *k* and then find the frequency estimates of $\alpha$, *a, b, c., h,* by equating first four theoretical frequencies with that of observed distribution.

From the observed distribution of post-partum amenorrhea period reported in Table 2, we equate the first four theoretical relative frequencies to observed distribution. The frequencies estimates come out to be $\hat{\alpha} = 0.6139$, $\hat{a} = 2$, $\hat{h} = 12.96$, $\hat{c} = 72.50$. The estimated frequencies based on these estimates are given in column 4 of Table 2. The fit given in the

TABLLE 2: OBSERVED AND EXPECTED DISTRIBUTION OE POST PARTUM AMENORRHEA FOLLOWING LIVE BIRTH (VARYING RISK OF EXPOSURE TO MENSES)

| *Duration of PPA* (in months) | *Number of mothers* | |
|---|---|---|
| | *Observed* | Expected for<br>v = 1 month<br>k = 3 months<br>a = 2.00<br>c = 72.50<br>h = 12.96<br>$\alpha$ = 0.6139 |
| 1 -3 | 98 | 98.00 |
| 3-5 | 90 | 90.01 |
| 5-7 | 60 | 60.04 |
| 7-9 | 38 | 37.99 |
| 9-11 | 27 | 26.84 |
| 11-13 | 25 | 20.45 |
| 13- 15 | 14 | 12.66 |
| 15- 17 | 8 | 7.88 |
| 17- 19 | 5 | 5.81 |
| 19-21 | 2 } | 3.48 } 15.13 |
| 21 + | 2 | 5.84 |
| Total | 369 | 369.00 |
| $\chi^2$ | 3.64 | |
| d. f | 5 | |

table is acceptable at 5 per cent level of significance tested by using Pearson's %. The ratio of the risk of resumption to menses for the two groups of mothers comes out to be 0.725, whereas it was 0.687 made in case of model I.

From the close look at both the fits obtained on the basis of frequency estimates of the two models, it is evident that the ratio of the risk of resuming menses of the mothers who do not supplement food to those who supplement food after at least 3 months of delivery comes out to be less than unity. We also observe that model II gives the better fit in comparison with model I.

Both the fits, discussed in Tables 1 and 2 support the use of some form of the modified exponential distribution to study the mechanism of variations in the duration of post-partum amenorrhea. Our aim in proposing the models has been to take account of a general distribution of post-partum amenorrhea under some simple assumptions regarding the lactation practices and study its effect on the risk of resumption of menses to women following different patterns of breast feedings. Though both the models discussed so far describe the data well, rr.cdel II is better and establishes its suitability in describing the distribution of the amenorrhea period. It appears that there is heterogeneity in the risk of resuming menstruation among women in the population.

Generally, reporting of post-partum amenorrhea period suffers from the errors of digit preference. The preferred digits are 6,12, and 18. This may be one reason that cell frequency corresponding to 11—13 is found inflated in the observed distribution (when we compare the observed and expected frequencies). The proposed distributions, therefore, may also provide a graduation formula to detect error in the data of port- partum amenorrhea. To make the model further realistic, it will be reasonable to apply a right hand traction on the distribution at a point of two years, (say), rather than allowing the variable to go upto infinity. Of course, this will make the distribution function complicated without much gain. The models discussed here may be treated as good approximation to the reality.

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