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Suddhendu Biswas* and Rita Jain"[1]

On a Reliability Approach to the Analysis of Interlive Birth Intervals and Parity Progression Ratios

Introduction

Analysis of interlive birth intervals and parity progression behaviour is of great importance in fertility analysis. The present work is in continuation of the approach of survival analysis or reliability application of much older demographic or actuarial technique in reliability analysis, has also suggested the possiblily or potential of reliability tools in exchange as a feedback for the survival or reliability analysis of demographic data through models.

The works of Feeney (1983) and Feeney and Ross (1984) are considered to be important contributions to parity progression analysis. Feeney's work was based on Lotka's stable population theory where age has been replaced by specific parity and interval since previous birth. He considered parity progression schedule, a combination of parity progression ratio and the birth-interval distribution for each birth order, to remain constant over a long period. Pathak and Ram (1989) further constructed fertility tables using age-specific fertility rates and parity-specific fertility based on the concept of life-table.

The present paper envisages to look into the phenomenon of parity progression from reliability or survival analysis point of view. This method has certain advantages over the traditional approaches in the sense that the parity progression ratios are estimable even without getting certain basic information such as parity-specific fertility rates, used by Pathak in the preparation of a life-table of parity progression.

Development of the Models

Pachal (1992) has considered the hazard rate of a further reconception at time t given by $\lambda_i(t)$ following the termination of $(i-1)$th order of conception ($i = 1, 2, \ldots, n$) as per the following schedule: .

$$\begin{aligned} \lambda_i(t) &= \lambda_i && , \theta \le t < t_{i1} \\ &= \lambda_i e^{\delta_i} && , t_{i1} \le t < t_{i2} \\ &= \lambda_i e^{\delta_i'} && , t_{i2} \le t < t_{i3} \\ &= \lambda_i e^{\delta_i''} && , t \ge t_{i3} \end{aligned} \tag{1}$$

* Department of Statistics, University of Dehi, Delhi-110 007.

where t_{i1}, t_{i2}, t_{i3} is the time limit (duration) of the ith birth after marriage in years.

This gives an intital uprise of hazard rate and then consistent decrease of the fertility behaviour at a given level in conformity with the fertility behaviour in earlier literature. It is known that a large number of women have their first child in first few years after marriage. So hazard rate is maximum at this period and diminishes subsequently.

Denoting the survival function of the conceptive delay $R_i(t)$, given as

$$R_i(t) = P[T_i \geq t] = 1 - F_i(t)$$

where $F_i(t)$ is the cumulative distribution function (c.d.f.) of the conceptive delay T_i (a r.v.), then

$$\begin{aligned} R_i(t) &= e^{-\lambda_i t} && ;\ 0 \leq t < t_{i1} \\ &= e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t - t_{i1})} && ;\ t_{i1} \leq t < t_{i2} \\ &= e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2} - t_{i1})} e^{-\lambda_i e^{\delta'_i}(t - t_{i2})} && ;\ t_{i2} \leq t < t_{i3} \\ &= e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2} - t_{i1})} e^{-\lambda_i e^{\delta'_i}(t_{i3} - t_{i2})} \\ & \quad e^{-\lambda_i e^{\delta'_i}(t - t_{i3})} && ;\ t \geq t_{i3} \end{aligned} \tag{2}$$

$$\Rightarrow F_i(t) = 1 - R_i(t) \tag{3}$$

$$\frac{d}{dt}(F_i(t)) = \frac{d}{dt} d\, R_i(t) = f_i(t) \tag{4}$$

Assuming a one-to-one correspondence between a conception and a birth, we get the estimates of interlive birth interval between $(i-1)$th parity and ith parity $(i = 1, 2, \ldots, 6)$ as

$$\begin{aligned} E(Ti) &= \int_0^{\alpha} R_i(t)\, dt \\ &= \int_0^{t_{i1}} e^{-\lambda_i t}\, dt + \int_{t_{i1}}^{t_{i2}} e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t - t_{i1})}\, dt \\ &+ \int_{t_{i2}}^{t_{i3}} e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2} - t_{i1})} e^{-\lambda_i e^{\delta'_i}(t - t_{i2})}\, dt \\ &+ \int_{t_{i3}}^{\alpha} e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2} - t_{i1})} e^{-\lambda_i e^{\delta_i}(t_{i3} - t_{i2})} e^{-\lambda_i e^{\delta''_i}(t - t_{i3})}\, dt \\ &= \frac{1}{\lambda_i}[1 - e^{\lambda_i t_{i1}}] + \frac{e^{-\lambda_i t_{i1}}}{\lambda_i e^{\delta_i}}[1 - e^{-\lambda_i e^{\delta_i}(t_{i2} - t_{i1})}] \end{aligned}$$

$$+ \frac{e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta i}(t_{i1}-t_{i1})}}{\lambda_i e^{\delta'_i}} [1 - e^{-\lambda_i e^{\delta'_i}(t_{i3}-t_{i2})}]$$

$$+ \frac{e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})} e^{-\lambda_i e^{\delta' i}(t_{i3}-t_{i2})}}{\lambda_i e^{\delta''_i}} \tag{5}$$

While the variance of the waiting time is given by

$$\mathrm{Var}(T_i) = 2\int^{\alpha} t R_i(t)\,dt - \left[\int^{\alpha} R_i(t)\,dt\right]^2$$

$$= 2\int_{o}^{t_{i1}} t e^{-\lambda_i t}\,dt + \int_{t_{i1}}^{t_{i2}} t e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t-t_{i1})}\,dt$$

$$+ \int_{t_{i2}}^{t_{i3}} t e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})} e^{\lambda_i e^{\delta_i}(t-t_{i2})}\,dt$$

$$+ \int_{t_{i3}}^{\alpha} t e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})} e^{-\lambda_i e^{\delta_i}(t_{i3}-t_{i2})} e^{-\lambda_i e^{\delta'' i}(t-t_{i3})}\,dt$$

$$- [E(T_i)]^2 \tag{6}$$

Further, the proportion of women not having further conception upto the expiry of the reproductive span t is given by

$$\pi = 1 - \Bigg\{ \Big[(1 - e^{-\lambda_i t_{i1}}) + e^{-\lambda_i t_{i1}} (1 - e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})})$$

$$+ e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})} (1 - e^{-\lambda_i e^{\delta'_i}(t_{i3}-t_{i2})})$$

$$+ e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})} e^{-\lambda_i e^{\delta'_i}(t_{i3}-t_{i2})} (1 - e^{-\lambda_i e^{\delta''_i}(t-t_{i3})}) \Big]$$

$$+ \Big[e^{-\lambda_i t_{i1}} (1 - e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})})$$

$$+ e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})} (1 - e^{-\lambda_i e^{\delta' i}(t_{i3}-t_{i2})})$$

$$+ e^{-\lambda_i t_{i1}} e^{-\lambda_i e^{\delta_i}(t_{i2}-t_{i1})} e^{-\lambda_i e^{\delta' i}(t_{i3}-t_{i2})}$$

$$(1 - e^{-\lambda_i e^{\delta'' i}(t-t_{i3})}) \Big] + \dots \Bigg\} \tag{7}$$

Thus parity progression probability will be $[1 - \pi]$.

Data and Application

The data utilized for testing the model relates to the Fertility Trend in Delhi.

A survey was conducted by Population Research Centre, Institute of Economic Growth, for Delhi Administration, Directorate of Health and Family Welfare in the year 1987.

The whole Delhi was divided into six zones. Each zone is further divided into several blocks. The blocks are chosen in the same manner as that considered in Census 1981, e.g., if one zone constitutes 30% of the population size, then sample size is determined accordingly. On the whole, a sample of 2114 households has been collected. The first house of the block is chosen by using random number table and thereafter every 10th house is picked up for collecting the information for all married couples in the house belonging to ages 45 and below.

A further sub-sample has been chosen by taking every fifth household of 2114 households already collected, so that a total of 415 households are covered.

Estimation of the Parameters of the Model

Since the models as described in (1) to (7) do not take account of the age of the women, heterogeneity emerges while fitting the data in the model for the estimation of the parameters. That is why, it has been found necessary to breakup the data for each parity according to the time of births (corresponding to that parity). For example, the distribution of the time of briths for first pairty is as given in Table 1.

TABLE 1: DISTRIBUTION OF WOMEN IN THE 1ST PARITY WITH RESPECT TO THE TIME OF BIRTH

The Time of First Birth after Marriage (in years)	*Number of Women*	*Cumulative Frequency*
0-1	176	176
1-2	83	259
2-3	42	301
3-4	21	322
4-5	11	333
5-«	8	341
6-7	6	347
7-8	3	350
8-9	4	354
9-10	0	354
10-11	1	355

From the above distribution percentiles were calculated. It was found 50% of women had their first child birth within 1.0181 years after marriage, 75% had within 2.1726 years and 90% had within 3.881 years. Assuming h = 1.0181, t_2 = 2.1726 and t_3 = 3.881 for i = 1 in (1), (2), (3), (4), (5) and (6), the hazard rate of births over points of time of the first parity has been worked out. This is given in Table 2.

TABLE 2: HAZARD RATE OF BIRTHS FOR FIRST PARITY

Duration (in year) x	Number of Women f	Cumulative Failures F	Failure density $f/l = f/355$	Hazard rate $\lambda(x)$
0-1	176	176	.4957	.6592
1-2	83	259	.2338	.6036
2-3	42	301	.1183	.56
3-4	21	322	.0592	.48
4-5	11	333	.0310	.40
5-6	8	341	.0225	.44
6-7	6	347	.0169	.5455
7-8	3	350	.0085	.46
8-9	4	354	.0113	1.33
9-10	0	354		-
10-11	1	355	.0028	2.0

These hazard rates were applied to estimate λ_i in the model, we get

$$\lambda_1(x) = \hat{\lambda}_i = .6592 \quad (0 \le t < 1.0181)$$
$$= \hat{\lambda}_1 e^{\hat{\delta}_1} = .48 \quad (1.0181 \le t < 2.1726)$$
$$= \hat{\lambda}_1 e^{\hat{\delta}'} = .4 \quad (2.1726 \le t < 3.881)$$
$$= \hat{\lambda}_1 e^{\hat{\delta}''} = .46 \quad (t \ge 3.881)$$

i.e.

$$e^{\hat{\delta}} = .7282$$
$$e^{\hat{\delta}'} = .6068 \tag{8}$$
$$e^{\hat{\delta}''} = .6978$$

A comparison of the observed mean and standard deviation of the interval between marriage and first birth is shown in Table 3, based on the data of Table 1 and from the model using (5), (6), and (8)

TABLE 3: COMPARISON BETWEEN OBSERVED MEAN AND STANDARD DEVIATION TO THE EXPECTED MEAN AND STANDARD DEVIATION

Mean Interval (in years)		S.D. of the Interval (in years)	
Observed	Expected	Observed	Expected
1.6634	1.8489	1.7235	2.095

Adjustment for the Period ofPost-partum Amenorrhoea (P.P.A.)

Further, to take account of post-partum infccundity period following each birth, a confidence limit for interlive birth interval has been computed by using the data collected by International Institute for Population Studies, Bombay.

The women followed were mothers, whose births were registered during January to March 1965 in four wards of the Bombay Municipal Corporation. These four wards were representative of the general population of Greater Bombay in terms of various socio-economic and demographic characteristics observed by the Census, 1961. A total of 1,535 women were contacted after the deliveries and subsequently followed up by repeated visits. For details of the survey design, see Karkal (1969).

A perusal of pattern of distribution according to duration of amenorrhoca reveals hetrogenceity in the distribution; particularly its bimodel characteristic, one mode lying around 4- 8 weeks of delivery and the other around 44-48 weeks. The study also exhibited the presence of two distinct sub-populations in the distribution of the pattern of amenorrhoea and showed that for 99 percent of the women in one group amenorrhoca terminated within 10 months after delivery, whereas almost none of the mothers in the counter group has resumed mensuration before completing a year (see Biswas 1973).

Taking the probability density of the period of the post- partum amenorrhoca as

$$f(x) = \pi \frac{e^{-x} x^{\theta - 1}}{\Gamma(\theta)} + (1 - \pi) \frac{(1 - \pi) e^{-x} x^{\theta - 1}}{\Gamma(\theta)}$$

$$\theta \leq x < \alpha ; \quad \theta = \theta$$

from the distribution of amenorrhoca of 1,174 women given in the appendix Table A-1 , one can get that either of the estimates of (θ, θ, π)

$$= (12.987756, 55.10602, \quad 0.7042)$$
$$(55.10602, \quad 12.987756, 0.2958)$$

is admissible and gives the same result with the same interpretaton. Taking $\theta = 12.99$, $\theta = 55.11$ and $\pi = 0.7042$, we conclude that about 70 percent of the women belonging to sub-population I have the mean period of amenorrhoea as 12.99 weeks, whereas the reamaining 30 percent in sub-population II have period of amenorrhoea as long as 55.10 weeks after delivery. The overall mean is 25.45 weeks of 0.48808 years.

After adding this post-partum amenorrhoca period to the mean interval, a comparison of the observed and estimated mean interval between ith and $(i + 1)$th birth is shown in Table 4 based on the data collected and from the model using (5), (6) and (8).

TABLE 4: OBSERVED AND ESTIMATED MEANS OF THE INTERLIVE BIRTH INTERVALS OVER DIFFERENT PARITIES

Parity	*Mean Interlive Birth Interval* (in years)	
	Observed	*Expected*
2	2.92468	2.79388
3	3.12448	3.35618
4	2.16368	3.73668
5	2.42108	4.25978
6	1.41948	3.33268

Analysis of the Parity Progression Rates

We have considered the data relating to the age of mother at the birth of the child corresponding to the parity and the time of occurrence of the birth from marriage in the analysis of parity progression ratios. The same is shown in Table 5.

TABLE 5: DISTRIBUTION OF WOMEN IN DIFFERENT PARITIES WITH RESPECT TO AGE AT THE TIME OF THE BIRTH OF THE CHILD

Parity 1			Parity 2			Parity 3			Parity 4			Parity 5			Parity 6		
Age Group	No. of W	Prop-ortion	Age Group	No. of W	Prop-ortion	Age Group	No. of W	Prop-ortion	Age Group	No. of W	Prop-ortion	Age Group	No. of W	Prop-ortion	Age Group	No. of W	Prop-ortion
15-20	50	.9464	15-20	5		15-20	3		20-23	4		22-25	4		24-26	5	
25-26	3		20-22	25	.881	20-22	11	.8824	23-25	13	.85	25-35	27	.7941	26-36	18	.75
26-28	3	.0536	22-25	30		22-25	22		25-34	38		35-38	3	.0882	36-38	1	.042
28-	0		25-29	14		25-31	27		34-36	4		38-	0		38-	0	
			29-30	4	.0476	31-33	3	.0441	36-38	1	.067						
			30-33	6	.0714	33-36	1	.0147	38–	0	.0167						
			33-	0		36-	1	.0147									
	56			84			68			60			34			24	

Using the model (7) and the observed proportion of women having births corresponding to the parity considered upto a certain age, the probability of not going to next parities till the end of the reproductive span has been worked out and shown in Table 6.

TABLE 6: THE PROBABILITY OF NOT GOING TO THE NEXT PARITY TILL THE END OF THE REPRODUCTIVE SPAN

Parity i	π = Probabilty of not going to the Parity (i + 1) till the end of Reproductive Span
1	$1-\{.9464 \times .9999 + .0536 \times .5168\} = .0260$
2	$1-\{.881 \times .9973 + .0476 \times .145 + .0714 \times .102\} = .1071$
3	$1-\{.8824 \times .9987 + .0441 \times .1314 + .0147 \times .0485\} = .1121$
4	$1-\{.85 \times .9983 + .067 \times .0629 + .0167 \times .0307\} = .1467$
5	$1-\{.7941 \times .9971 + .0882 \times .0706\} = .2020$
6	$1-\{.75 \times .9927 + .042 \times .0058\} = .2552$

Based on the data of Table 6, a life table on the parity progression has been constructed and shown in Table 7.

TABLE 7: LIFE TABLE ON THE PARITY PROGRESSION

x	he	dx	1"	P'	Lx	Tx	ex°
1	1000	26	.0260	.9740	987	4037.70	4.04
2	974	104.3	.1071	.8929	921.85	3050.70	3.13
3	869.7	97.5	.1121	.8879	820.95	2128.85	2.45
4	772.2	113.28	.1467	.8533.	715.55	1307.90	1.69
5	658.9	133.1	.2020	.7980	592.35	592.35	0.89
6	525.8	134.2	.2552	.7448			

As mentioned earlier it may be noted that a similar life table of parity progression was constructed by Pathak and Ram (1989) using-the estimates of parity-specific fertility rales. A comparison between the parity progression rates shown in Table 7 and that of Pathak (1989 : 350, Table 4) is shown in Table 8.

TABLE 8: A COMPARISON OF PARITY PROGRESSION RATES

Parity	*Palhak*	*Present Studies*
1-2	.8934	.9740
2-3	.8059	.8929
3-4	.7393	.8879
4-5	.6188	.8533
5-6	.3368	.7980
6-7	.4582	.7448

The comparison shows that Paihak's estimates are consistently underestimate in comparison to our present investigation. However, the difference found may be due to the difference in intrinsic behaviour of fertility in the two populations considered.

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APPENDIX

TABLE A-l : FITTING OF A MIXTURK OF I DISTRIBUTION TO Till- FMI'IKICAI.
DISTRIBUTION OF AMKNORRIIOHA OF 1174 WOMIiN

Period of amenorrhoea (in weeks)	*Observed frequency*	*Expected frequency*
0-4	66	65.19
4-8	178	171.57
8-12	163	185.97
12-16	136	165.28
16-20	101	108.57
20-24	65	64.55
24-28	55	37.76
28-32	31	25.75
32-36	35	23.60
36-40	47	26.92
40-44	32	32.29
44-48	61	68.S4
48-52	38	38.28
52-56	37	38.08
56-60	42	34.63
60-64	30	29.97
64-68	22	24.47
68-72	17	19.12
72 and above	18	13.16
Toul	1174	1174.00

***Source:* (Biswas 1973 : 211).**