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S. Mukherjee*, K. K. Singh, C. M. Suchindran** and B. N. Bhattacharya***

A Probability Distribution for Last Closed Birth Interval

Introduction

IN many retrospective surveys on fertility and family planning, conducted in different parts of the world, a common practice has been to collect information on birth histories of married women. The availability of such data has caused demographers to conduct birth interval analysis with several objectives in mind: (1) to understand the biological process of family formation through mechanisms consistent with the observations; (2) to estimate parameters of proximate determinants of fertility; (3) to investigate the possible consequences on various alterations in these parameters; and (4) to construct indices for measuring changes in fertility in the context of the implementation and evaluation of family planning and maternal child health care (MCH) programmes.

Information on closed birth intervals related to births in the distant past is subject to lapses of memory and other sources of bias that often depend on the recall period. In recent years retrospective enquiries of data collection have been confined to a particular period preceding the reference date of a survey. Special attention has been paid to collect information on the last closed interval. The last closed interval requires recall of information only on the two most recent births before the survey date and is expected to be more reliable than earlier closed intervals. Here we develop a model to analyse the last closed birth interval with the goal of estimating biological parameters of human fertility.

The socio-cultural practices relating to timing and frequency of coitus, the practice of voluntary or involuntary abstinence after childbirth, and other factors may influence birth spacing. Abstinence from coitus is a culturally patterned norm, scrupulously observed for various reasons by most people in rural India and other traditional societies in different parts of world (Mahadevan 1979; Santow 1978). Sexual taboos are observed with varying degrees of strictness in different societies because women are considered to be unclean and polluting during the first few months postpartum. The period of abstinence is determined by some marker of the infant's development—sitting, crawling, walking, or cutting teeth. Intercourse

* Population Studies Unit, Indian Statistical Institute, Calcutta-700 035.
** Carolina Population Center, The University of North Carolina at Chapel Hill, CB # 8120, University Square, Chapel Hill, N. C. 27516-3997, USA.

with a lactating woman is also considered an unethical practice in some societies; this taboo may prolong the period of abstinence considerably in societies where breast-feeding is of long duration. Previous attempts at modeling birth intervals considering the risk for conception for a single woman to be constant within a birth interval are not appropriate for such situations.

Poole (1971), Sheps and Menken (1972), and others have outlined procedures to obtain theoretical distributions of last closed intervals of a specific order. In addition to parity, these distributions depend on age and/or on marital duration. The derivations of the models under a set of assumptions requires the knowledge of the distributions (or their functional forms) of the earlier birth intervals. Expressions often become too complex, thus, making their applications difficult. Here, we attempt to derive a distribution of last closed birth interval occurring during a specific period. The model is not parity dependent. This feature makes it simple to apply and avoid the problems of errors in parity data. The model is also applicable in situations where abstinence following childbirth and taboos relating to coital frequency during the early part of interval are widespread. The model is illustrated with the data collected from two Indian fertility surveys.

Model

Suppose married women of the same age, say b, are sampled at some time T'' and the dates of the last two births if they occurred during the preceding T' years of the survey are recorded. The distribution of the length of the last closed interval for women who have such an interval interior to age interval $(a, b), (a-b-T')$ or time period $(T''-T, T'')$ is obtained under the following assumptions:

(1) $(T'' - T')$ is a distant point after marriage and the parameters of reproduction had been constant for a considerable period prior to age "a" so that equilibrium is attained at $(T'' - T')$. The parameters also remain the same during the period $(T'' - T', T'')$.

(2) The duration of postpartum amenorrhoea (PPA), say, U, and the period of sexual abstinence, say, V, following a live birth are independently distributed, non-negative, random variables with corresponding distribution functions $G(t)$ and $H(t)$. The distribution of the non-susceptible period, Z, associated with a live birth is given by

$$Z = \max(U, V).$$

(3) For a woman with $Z = z$ and $V = v$, the conditional instantaneous risk of conception—the one immediately following a live birth after an elapse of time / is

$$m(t/v);\ t > z$$

$$m_0 = \lim_{t\to\infty} m(t/v);\ \text{for all } v.$$

Since coitus resumes after the period of abstinence, its frequency and consequently $m(t/v)$ is assumed to depend on the duration of postpartum abstinence period and t, until a conception occurs or the normal level is attained, whichever is earlier.

(4) 6 is the probability that conception results in a foetal loss, $0 < \theta < 1$.
(5) The length of the non-susceptible period comprising the duration of pregnancy and PPA associated with foetal loss is an exponentially distributed random variable with mean l/c, $c > 0$.
(6) The conditional instantaneous risk of conception following the termination of the non-susceptible period following a foetal loss is m_0.

Functional forms of $G(t)$, $H(t)$, $m(t/v)$ and constants involved therein and parameters 0 and c do not change with age and parity in the interval $(T'' - T', T'')$.

Derivation

Let us consider a woman with $U= u$, $V= v$ and thus $Z = z$ where $z = \max(u, v)$. Measuring the time since $(T'' - T')$, denoted by S_t ,S_2,... the time of recording the successive births which occurred after $(T''\sim T')$;$X_t=S_t$ and$X_i =S_i-S_{i-1}$ $(i =2,3,..)$. Since the reproduction process was in equilibrium at time $(T'' - T'')$, the distribution of the time of first recording for the woman is

$$K^*(t) = \int_0^t [\{1 - K(x)\}/\mu]\, dx \tag{1}$$

where $K(x)$ is the distribution function of closed birth interval and \i is the mean length of the closed birth interval and is given by (see Sheps and Menken 1973)

$$\mu = \int_0^\infty \{1 - K(y)\}\, dy. \tag{2}$$

The last closed interval within the period $(T'' - T, T')$ could be observed only for those women who have had two or more births during the period.

The distribution function of the length of the last closed interval for a woman who has experienced such an interval during $(T''- T, T'')$ [i.e., delivered two or more births during $(T''-T',T'')$]is

$$B(t) = \frac{\Pr\,[\#\text{ of women with last closed interval} \le t \text{ and atleast two births during } (T'' - T', T'')\,]}{\Pr\,[\#\text{ of women with at least two births during } (T'' - T', T'')\}}$$
(3)

The proportion of couples who have had exactly i $(i \geq 2)$ births during $(T'' - T', T'')$ and the length of the interval between $(i-1)^{\text{th}}$ and i^{th} births is smaller or equal to t is

$$\Pr\,[\, X_i \le t \cap S_i \le T' \cap S_{i+1} > T'\,]\ . \tag{4}$$

Thus, the proportion of couples with length of the last closed interval lying between t and $t + dt$ is

$$B^*(t) - \sum_{i \geq 2} \Pr [\, S_{i-1} \leq T' - t \cap S_{i-1} + X_{i+1} > T' - t \,]\, k(t)dt, \qquad (5)$$

where $k(t)$ is the density function corresponding to distribution function $K(t)$. Since (X_i; i = 2, 3, . . .) are identically and independently distributed, therefore

$$\begin{aligned} \Pr [\, S_{i-1} \leq T' - t \cap S_{i-1} + X_{i-1} > T' - t \,] &= \Pr [\, S_{i-1} \leq T' - t \cap S_{i-1} + X_i > T' - t \,] \\ &= \Pr [\, S_{i-1} \leq T' - t \cap S_i > T' - t \,] \\ &= \mathrm{P}_{i-1} (\, T' - t \,), \end{aligned}$$

where P_{i-1} ($T' - t$) is the probability of experiencing exactly (i – 1) births during ($T'' - T'$, T''). The equation (5) becomes

$$B^*(t) = \sum_{i \geq 2} P_{i-1}\, (T' - t)\, k\,(t)\, dt$$

which reduces to

$$B^*(t) = \Pr[S_1 \leq T' - t]\;\; k(t)dt. \qquad (6)$$

The proportion of women with two or more births during ($T'' - T'$, T'') is

$$\int_0^{T'} B^*(t)\, dt.$$

Thus, the distribution function of the length of the last closed interval for a woman who has experienced such an interval in ($T'' - T'$, T'') is written as

$$B\,(t) = \frac{\int_0^t B^*\,(t)\, dt}{\int_0^{T'} B^*(t)\, dt} \;\; ; \; 0 < t < T' \qquad (7)$$

Under the assumption (2) of the model, the distribution of last closed birth interval can further be formulated as follows:

The duration of postpartum abstinence will be the period of non-susceptibility when PPA is less than or equal to the period of abstinence. In this case the probability that Z will lie in the interval (z, z + dz) is

$$G(z)dH(z).$$

Again, the duration of non-susceptibility will be the duration of PPA when the period of abstinence is less than the duration of PPA. Thus, the probability that Z and *V* will lie in the intervals $(z, z + dz)$ and $(V, v + dv)$ respectively $(0 < v < z)$, is

$$dG(z)dH(v).$$

Thus, the proportion of women in the population with last closed interval lying between t and $t + dt$ is

$$B^* (t) = \int_{(0,t)} G(z)\, dH(z) \Pr [\, S_1 \le T' - t/z, z\,]\, k(t/z, z)\, dt + \int_{[0,t]} dG(z) \int_{[0,z)} dH(v) \Pr [S_1 \le T' - T/v, z\,]\, k(t/v, z\,)\, dt. \qquad (8)$$

Substitution of equation (8) in equation (7) will give the appropriate distribution.

Illustration of the Model

Data

Data used for illustration of the model came from two surveys: (1) "Rural Development and Population Growth — A Sample Survey", carried out in *1978* with financial support from the University Grants Commission (UGC), and (2) "Effects of Socio-Economic Factors on Determinants of Fertility in Eastern Uttar Pradesh", carried out during 1987-88 with financial support from the Indian Council of Medical Research (ICMR). Both were conducted by the Centre of Population Studies, Banaras Hindu University (CPS, B.H.U.), India.

One common objective of the surveys was to obtain reliable data relating to fertility to study the socio-economic and behavioural factors affecting fertility and estimation of the bio-cultural parameters of reproduction in Eastern Uttar Pradesh.

In the UGC S urvey stratified random samples of 19 villages were selected from Varanasi tehsil and adjoining areas and in the ICMR survey a sample of 19, 8, and 29 villages was selected respectively from rural areas of Varanasi, Ghazipur, and Azamgarh districts of Eastern Uttar Pradesh. The surveys included all households from selected sample villages numbering 3514 and 3931 in UGC and ICMR surveys respectively. A couple was defined eligible if both partners were alive on the reference date of the survey and the wife was less than age 50. In addition to other information, data relating to births that occurred to eligible women during the preceding seven years from the reference date of survey were collected in the ICMR study while the UGC study collected the complete birth history.

The last closed birth intervals of women in the current age groups 25-30, 30-35, and 35-45 whose effective marriage duration was 12 years or more and who delivered two or more births during the preceding seven years from the reference date are examined in this study.

Only women having marital durations of 12 years or more on the reference date of survey were considered becaus'e the reproduction process may be assumed to have attained equilibrium by that time. Among the eligible couples, only those who did not adopt any terminal method of family limitation and who were both usual residents of the village were included in the study. In ICMR study information on the status of women (menstruating, pregnant, amenorrheic, menopausal) on the reference date was obtained and those who had reached menopause were also excluded.

Application

Application of the model requires information on the distributions of PPA and the period of abstinence, T_1 — the time beyond abstinence during which coital frequency depends on time, 9 — the incidence of foetal loss, and 1/c mean duration of the non-susceptible period associated with a foetal loss.

Analyses of data on PPA from surveys in Eastern Uttar Pradesh and as well as data from rural regions of India and Bangladesh, where extended breast-feeding was the practice of most women, reveals that the distribution of PPA has two modes, one within few months after birth and another many months later (Ford and Kim 1987; Singh 1990; Singh and Bhaduri 1971). Because of this observation, in the specification of the model, we assume two groups of women whose PPA takes two values t_1 and t_2 with probabilities p_1 and p_2 respectively, $0 < p_1 \leq 1$, $p_1 + p_2 = 1$. On the basis of the analysis of data on PPA from the ICMR survey, t_1 and (2 are taken as 0.1 year and 1.00 year respectively and the values of p_1 are taken as 0.60,0.50, and 0.45 for women belonging to age groups 25-30,30-35, and 35-45 respectively.

In the study area, some taboos on commencement and frequency of coitus after childbirth are prevalent. The study on Breast-Feeding and Its Effects on Fertility was carried out in rural and urban areas of Varanasi in 1987-89. In rural areas, the study was conducted in five villages also covered by the ICMR survey. In this survey, information on duration of abstinence and PPA was collected. For women with PPA durations of 1-2 months, 2-3 months, 3 -4 months, and 4 months or more, the means of the duration of effective abstinence (which is practised after resumption of menstruation) were approximately 2,1.8,0.7 and 0.2 months respectively. Figures indicate that women usually abstain from intercourse on an average 3 months after childbirth.

Thus under the assumptions (1) the duration of post-partum abstinence is the same for all women and it is of length τ, (2) the conception rate $m(t/\tau)$ is a polynomial of degree τ in t for $\tau < t \leq \tau + T_1$ and constant thereafter and is of the form

$$m(t/\tau) = \begin{cases} \sum_{j=0}^{r} q_j (t-\tau)^j & \text{for } z < t \leq \tau + T_1 \\ \sum_{j=0}^{r} q_j T_1^j & \text{for } t > \tau + T_1 . \end{cases} \tag{9}$$

Under these assumptions the expression for $B^*(t)$ in (8) reduces to

$$B^*(t) = \sum_{i=1}^{2} p_i \int_0^{T'-t} [(1 - K(x/i))/\mu_i] \; dx \, k(t/i). \tag{10}$$

where $K(t/i)$ is the distribution function of the closed birth interval for a female with $V = \tau$ and $U = t_i$. The density function and mean length of the interval corresponding to $K(t/i)$ are $k(t/i)$ and μ_i respectively, where

$$K(t/i) = \sum_{j=0}^{2} A_j K(t/i) \qquad \text{(see Appendix)}. \tag{11}$$

Denote $M(t/i) = \int_{z_i}^{t-g} m\,(x/\tau)\,dx$, $Z_i = \max\,(t_i\,,\tau)$, $(0 \le Z_i < t-g)$, $h_i = t_i + g$, $i = 1, 2$ and $h' = \tau + g$. Then

$$K_0\,(t/i) = 1 - \exp\,(-M(t/i))\,. \tag{12}$$

Substituting in equation (12) for $m\,(t/\tau)$, $M(t/i)$ reduces to the following:
For $h' \geq h_i$

$$M(t/i) = \begin{cases} \varphi(t) & ; \text{if } h' < t \le h' + T_1 \\ \varphi(T_1 + h') + \left(\sum_{j=0}^{r} q_j T_1^j \right) (t - h' - T_1) & ; \text{if } t > h' + T_1 \end{cases} \tag{13}$$

for $h' < h_i \le h' + T_1$

$$M(t/i) = \begin{cases} \varphi(t) - \varphi(h_i) & ; \text{if } h' < t \le h' + T_1 \\ \varphi(T_1 + h') - \varphi(h_i) + \left(\sum_{j=0}^{r} q_j T_1^j \right) (t - h' - T_1) & ; \text{if } t > h' + T_1 \end{cases} \tag{14}$$

and for $h_i > h' + T_1$

$$M(t/i) = \sum_{j=0}^{r} q_j T_1^j\,(t - h_i) \;\; ; \text{if } t > h_i\,, \tag{15}$$

where

$$\varphi(t) = \sum_{j=0}^{r} \frac{q_j}{j+1}\,(t - h')^{j+1}\;.$$

Similarly, $K_j(t/i)$, $(J = 1, 2)$ can be obtained from expression (A4).

Maximum likelihood estimation procedure was used to obtain the parameters of the model. The Newton-Raphson iteration procedure was used to solve the equation. In estimation of parameters, it was assumed that $\theta = 0.15$, $1/c$ equal to 0.5 year, t equal to 0.25 year and T_1 equal to 2 years. When $m(t/t)$ is constant, the distribution involves only one

TABlE: 1 : VALUES OF THE CHI-SQUARE STATISTICS TO TEST THE DIFFERENCE BETWEEN OBSERVED AND EXPECTED FREQUENCIES OF LAS'1' CLOSED BIRTH INTERVAL FOR DIFFERENT FORMS OF RISK FUNCTION

Name, of Survey	*Risk Function*	*Age of women on reference date of survey* (in years)					
		25-30		30-35		35-45	
		X^2	*D.F.*	X^2	*D.F.*	X^2	*OF.*
ICMR	Constant	6.8	5	17.7	5	13.9	5
	Linear	5.1	4	4.5	4	4.7	4
	Quadratic	4.9	3	3.6	3	2.4	3
	LR1*	2.4		14.4		9.7	
	LR2**	0.2		0.9		2.2	
UGC	Constant	45.4	5	58.2	5	38.3	5
	Linear	23.0	4	12.7	4	9.8	4
	Quadratic	3.5	3	2.4	3	8.6	3
	LR1*	25.1		45.1		28.7	
	LR2**	19.3		9.9		2.3	

* is log -likelihood ol constant to linear.

** is log-likelihood of linear to quadratic.

TABU; 2 : DISTRIBUTION Of WOMEN ACCORDING TO THE LENGTH OE LAST CLOSED INTERVAL BY PRESENT AGE (UGC SURVEY)

Last closed interval (in years)	*Age of women on reference dale of survey (in years)*					
	25-30		*30-35*		*35-45*	
	Risk Function					
	Quadratic		*Quadratic*		*Linear*	
	Observed Frequency	*Expected Frequency*	*Observed Frequency*	*Expected Frequency*	*Observed Frequency*	*Expected Frequency*
0.-1.25	13	11.2	9	8.4	6	6.7
1.25-1.75	12	15.4	16	17.1	28	21.6
1.75-2.50	88	84.4	114	113.6	110	126.0
2.50-3.25	96	102.4	135	130.8	122	103.4
3.25-4.00	48	40.6	56	60.8	50	54.4
4.00-4.75	11	13.1	20	23.0	19	25.4
4.75-7.00	4	4.9	14	10.3	17	14.5
Total	272	272.0	364	364.0	352	352.0

TABLE 3: DISTRIBUTION OF WOMEN ACCORDING TO THE LENGTH OF LAST CLOSED INTERVAL BY PRESENT AGE (ICMR SURVEY)

Last closed interval (in years)	*Age of women on reference date of survey (in years)*					
			25-30		*35-45*	
Risk Function						
Linear			*Linear*		*Linear*	
	Observed Frequency	*Expected Frequency*	*Observed Frequency*	*Expected Frequency*	*Observed Frequency*	*Expected Frequency*
0-1.25	11	12.2	19	14.9	6	7.1
1.25-1.75	31	25.9	29	36.0	24	17.8
1.75-2.50	56	62.2	120	120.2	81	90.2
2.50-3.25	47	47.7	98	93.0	81	73.5
3.25-4.00	31	30.4	48	52.7	42	43.1
4.00-4.75	24	17.7	32	26.9	19	22.4
4.75-7.00	10	13.9	15	17.3	16	14.9
Total	210	210.0	361	361.0	269	269.0

TABLE 4 : ESTIMATES OF THE PARAMETERS. VARIANCES. AND CORRELATION COEFFICIENT BETWEEN ESTIMATORS

Estimates	*Name of Survey*					
	ICMR			*UGC*		
	Age of women on reference date of survey (in years)					
	25-30	30-35	35-45	25-30	30-35	35-45
q_o	0.399	0.350	0.304	0.770	0.366	0.193
q_1	0.123	0.290	0.279	-1.111	-0.309	0.462
q_2	—	—	—	0.853	0.448	—
$v(q_o)$	0.006	0.003	0.004	0.051	0.019	0.003
$V(q_1)$	0.013	0.005	0.006	0.223	0.097	0.004
$v(q_2)$	—	—	—	0.055	0.025	—
$corr(q_0,q_1)x(-l)$	0.692	0.831	0.869	0.898	0.881	0.863
$Corr(q_0, q_2)$	—	—	—	0.752	0.739	—
$Corr(q_1,q_2)x(-l)$	—	—	—	0.953	0.752	—

parameter q_0 The Initial value of q_0 can be obtained by equating the observed mean of the last closed interval to its theoretical value When $m(t/t)$ is a polynomial of degree one, there are two parameters q_0 and q_1 — maximum likelihood estimates of q_0 obtained for constant form of $m(t/t)$ and zero were taken as initial values of q_0 and q_l respectively Similarly q_0 andq_1, obtained above and zero were taken as the initial values of q_0, $q1$, and q_2 respectively, when $m(t/t)$ is quadratic

In Table 1 we give the fit of the model taking constant, linear, and quadratic form of risk function. The constant form of hazard function gave a poor fit. The distribution worked well with the linear form of risk function for the ICMR data and with the quadratic form of risk function for the younger ages for the UGC data.

The expected distributions of women according to the length of last closed interval by age appear in Tables 2 and 3.

Estimates of parameters, variance of estimators, and correlation coefficients between estimators appear in Table 4

For 1978 data, the best fitting model showed the conception rate $m(t/t)$ during the interval $(t, t + T_1)$ to be quadratic of the form $q_0 + q_l(t - t) + q_2(t - t)^2$ for age groups 25-30 and 30-35 (Table 2) Contrary to expectation the value of q_1 was found to be negative, and the value of q_2 was positive. These parameter values indicate that $m(t/t)$ decreases during the interval $[(t, t - (q_l/2q_2)]$ and increases thereafter more rapidly till time $t + T_1$. However, the width of the interval $[t, t - (q_l/2q_2)]$ is small and found to range between 0 to 4 months for the two age groups. Also the rate of decrease in $m(t/t)$ is slow This deviation from the preconceived pattern of $m(t/t)$ may perhaps be due to the simplifying assumptions made about various components. For the age group 35-45, the linear form of $m(t/t)$ gave adequate tit The estimated risk of conception at time t, $m(t/t)$ of the women who belong to younger age group is higher than that of females who belong to the older age group. By plotting $m(t/t)$ (not shown) against t of women aged 25-30, 30-35, and 35-45 at the reference date of the survey, it can be seen that the curve of $m(t/t)$ for age group 25-30 lies above that ot the other two age groups

For 1987 data, the linear form of $m(t/t)$ function gave adequate fit for all age groups (Table 3) The likelihood ratio criterion for testing the null hypothesis $q_2 = 0$ has been calculated for each set of data and found to be insignificant. Thus, we present the estimate of parameters taking the linear form of risk function in the tables. The estimate of the conception rate at the start of cohabitation, q_0 is highest for women whose current age is 25-30 and, q_1, a measure of the rate of increase, is lowest for the same age group of women Thus the age group 25-30 shows high conception rate in the beginning of the interval and subsequently with time, falls below the other two groups.

The adequacy of the linear form of risk function for the age groups 25-30 and 30-35 for 1987 data as compared to the quadratic form of risk function for 1978 data seems to indicate that recently there is some relaxation in sexual restrictions on couples in the study area This result indicates the sensitivity ol the model parameters to changes in bio-social factors affecting human fertility

The instantaneous risk of pregnancy following the previous birth is assumed to increase during $(t, t + T_1)$ and attains a plateau thereafter In fact both t and T_1 j may depend on survival status of the child, breast-feeding practices, demographic characteristics (such as age, marital duration, parity, number of surviving children, and their age and sex), and cultural characteristics, and may also vary among women having similar characteristics. The variation in *m(t/i)* among women may be incorporated in the present model in a manner similar to that discussed by Bhattacharya *et al.* (1988).

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APPENDIX

The length of an interval between two consecutive births for a female with $U = u$, $V = v$, and $Z = \max(U, V)$ could be decomposed as

$$T/n = Z + \sum_{j=0}^{n} W_j + \sum_{j=1}^{n} Y_j + g$$

given that there had been exactly n intervening foetal wastages, where

W_j: The duration of stay in a fecund state until conception following termination of the non-susceptible period of the live birth at the start of the interval ($j = 0$) or following j^{th} foetal loss in T' ($j \geq 1$),

Y_j: The duration of non-susceptibility associated with the j^{th} foetal loss, and

g: The period of pregnancy associated with the live birth.

Here if, $(Z + W_o)$, W_1, $W_2, \ldots, Y_1, Y_2, \ldots$ are statistically independent and for each $j \geq 1$, W_j and Y_j are exponentially distributed with parameters m_o and c then, it is easy to see that the Laplace transform of the unconditional distribution function of the random variable T is

$$\psi(s) = \sum_{n=0}^{\infty} (1-\theta)\,\theta^n F^*(s) \left\{m_0/m_0 + s\right\}^n \left\{c/c+s\right\}^n \exp(-sg)\,,$$

where $F^*(s)$ is the Laplace transform of the distribution function of the random variable $(Z + W_o)$.

The above Laplace transform is rewritten as

$$\psi(s) = F^*(s)\,[A_0 + A_1\{v_1/v_1 + s\} + A_2\{v_2/(v_2 + s\}]\exp(-sg), \text{ where}$$

$$A_0 = 1 - \theta\,,\; A_1 = \frac{m_0\, c\theta\,(1-\theta)}{v_1\,(v_2 - v_1)}\;,\; A_2 = \frac{m_0\, c\theta\,(1-\theta)}{v_2\,(v_1 - v_2)}$$

v_1 and v_2 are the additive inverse of the roots of the equation $s^2 + (m_0 + c)s + m_0\, c\,(1-\theta) = 0$ and v_1 and v_2 are non-negative and distinct.

The inverse of $\Psi(s)$ — the completed distribution of closed birth interval T is

$$K(x) = \sum_{j=0}^{2} A_j K_j(t) \qquad \text{(A2)}$$

$$\text{where} \quad K_0(t) = 1 - \exp\left\{-\int_{x}^{t-g} m(x/v)\,dx\right\};\; t > z \qquad \text{(A3)}$$

$$K_j(t) = \int_{z}^{t} K_0(y)\, v_j \exp\{-v_j\,(t-y)\}\,dy\,;\; t > z\; (j = 1, 2)\,. \qquad \text{(A4)}$$