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On Lutz and Scherbov's Sensitivity of Aggregate Life Expectancy to Different Averaging Procedure — A Research Note

The Problem

IN a very interesting paper Lutz and Scherbov (1992) have discussed the problems resulting from alternative ways of combining the period life tables of different populations. They begin by pointing out that one result is obtained for the global life expectancy when it is computed byweighting the country life expectancies by the respective number of births and another result is obtained when it is derived from the would age-specific death rates. Based on the 1980-85 period life tables, the later was found to be greater than the former by as much as 2.5 years (United Nations, 1989, 1991). Similarly, significant differences between life expectancies may also be found when the male and female life tables are combined by following the aforementioned procedures. In addition, the life tables combined by the latter method may also yield a life expectancy lying outside the range of the life expectancies, of the component life tables. It is easy to see that such is not the case when the former method (weighting by births) is used to generate the life expectancy. The authors have attributed the problem associated with the latter to the computation of an average by a nonlinear method (a definition of a non-linear average would have been helpful to the readers). It may be seen that the cohort life tables do not present this problem and the former method treats the component period life tables as cohort life tables albeit artificial. The latter method on the other hand is based on mortality experiences of different cohorts which may not constitute a stationary population. Consequently, the joint life expectancy produced by this method is not a weighted average of any sort of the component life expectancies and is not constrained to lie somewhere in the range defined by the minimum and the maximum of these expectancies. The authors have empirically demonstrated this phenomenon by choosing two stable populations one of which has a negative while the other has a positive intrinsic rate of growth. They have not specified the relative sizes of the two populations at the time of merger, although it appears that the sizes were taken to be the same. They have also not mentioned that the choice of sizes would have a significant influence on the results. Thus, this demonstration by simulation of the circumstances when

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the joint life expectancy may or may not lie inside the range of component life expectancies has only limited application.

The Effect of Weighting

In the following we shall take a closer look at the computational steps of the joint life expectancy by merging the distributions of populations and deaths by age. For simplicity, we shall use the example of two populations denoted by A and B. For the merged populations, the age-specific death rate of people aged x may be written as

$$m(x) = w(x)m(x,A) + (l-w(x))m(x,B), 0 \leq w(x) \leq l \quad . \tag{1}$$

where m(x,A), m(x,B) and m(x) stand for the age-specific death rates at age x of population A, population B and the combined population respectively and *w(x)* is the proportion of x year olds in the combined population that belongs to population A.

Multiplying both sides of (1) by the size of the merged stationary population L(x) and rearranging terms, we get

$$d(x) = \frac{w(x)L(x)}{L(x,A)} d(x,A) + \frac{(1-w(x))L(x)}{L(x,B)} d(x,B) \tag{2}$$

where d(x) stands for the number of deaths at agex in the merged population and definitions of the other terms are similarly straightforward. Compressing (2) as

$$d(x) = c(x,A)d(x,A) + c(x,B)d(x,B) \tag{3}$$

we observe that it is a linear compound of the ds in which the weights c(x, A) and c(x,B) do not necessarily add up to one. Similarly, the relationship between the average ages at death of the component and the merged populations can be expressed as

$$e(0) = c(A)e(0,A) + c(B)e(0,B) \tag{4}$$

where c(A) and c(B) are some sort of averages over age of c(x,A) and c(x,B) respectively and like them do not necessarily add up to one either.

It is obvious from (4) that the relationship among the life expectancies is not nonlinear as Lutz and Scherbov have noted in their paper but is is very much linear with weights that do not necessarily add up to one. Accordingly, e(O) is not constrained to lie in the interval determined by the life expectancies of the component populations. Incidentally, it may be mentioned that when the radixes of the life tables are proportional to the number of births or any other constant number, the c(x)s are constants, c(x,A) and c(x,B) are the same for all *x* and they add up. to one. The same is the case with c(A) and c(B) and accordingly, e(O) will always lie in the interval determined by e(O,A) and e(O,B) and accordingly, e(O) will always lie in the interval determined by e(O,A) and e(O,B).

Discussion

Be that as it may, Lutz and Scherbov have presented and discussed an important issue concerning the procedure of combining life tables. Setting aside the well known distinction between the cohort and the period life tables, the fact remains that any life table is nothing but an abstraction that tells its users how a cohort, real or hypothetical, lives and dies in specific time intervals. Such a picture is drawn using only a set of age-specific mortality rates and nothing else. Of course, how those rates are derived becomes an important issue only when the life tables are expected to provide clearcut answers to specific questions. If for example, in a closed population, one wants to know the probability of a person's surviving to a specific age according to the pattern of mortality prevailing in a certain period of time, the age-specific death rates for the total population may provide the answer. If on the other hand one wants to express this probability in terms of the conditional probabilities of survival of say males and females, we will certainly need not only the same data sets separately for the two sexes but will also need a consistent weighting scheme which can only be provided by the sex ratio at birth. Similarly, if the population eventually becomes stationary while subject to the continuation of a given schedule of sex-age specific death rates, the age distributions of the two sexes must be weighted by the respective number of births.

The point is that the two types of life tables will always show differences in their estimates of life table functions unless the component populations are stationary to begin with. One is affected by the age compositions of and the other by the number of births in the component populations and both are true in their respective places. Interestingly enough, the latter can, in some sense, be shown to be based on the former. Consider for example, the male age-specific death rates. These rates can certainly be regarded as those obtained by merging all the male populations belonging to different region and/or ethnic and/or socioeconomic status and/or other categories. In all likelihood, these sub-populations will have different age compositions as well as different age-specific rates. That is to say, in almost all populations, the males or the females as a group, is quite heterogeneous in the sense that the chances of survival at any age are likely to differ widely among different categories in which that population can be subdivided. Insistance of weighting the life expectancies may not therefore be justified unless the population is subdivided to the point where the sub-population can be regarded as homogeneous with respect to their respective levels of mortality. Therefore, if it is appropriate to compute the male or female life expectancy from the respective age-specific rates, it is similarly appropriate to follow the same procedure for the total population. An extension of this logic justifies the latest revision by the United Nations to compute the global life expectancy by merging all the countries. Nevertheless, the fact remains that as a measure of the level of mortality, life expectancy, like any other measure depends on the nature of the data set from which it is derived. The justification of using one set over the other is to be derived from the statement of purpose and from that alone. Since, life-expectancy has been found to be quite sensitive to this choice, it seems proper that serious considerations be given to the advisability of computing it by alternative methods when the necessary data are available. Not only will such a strategy serve people with different interests but a detailed examination of the differences among the estimates may also turn out to be a stimulating and worthwhile exercise.

Concluding Remarks

The similarity between the current paradox and the age old controversy about the two sex stable population model should not go unnoticed. In some ways, the latter can be shown to include the former as a particular case. Take for example the male and the female stationary populations. If the prevailing mortality rates continue to operate and the fertility rates are such that the net reproduction rate is equal to one, the eventual age composition of the combined male and the female population can be obtained by summing the two life tables with weights determined by the sex-ratio at birth. In the intervening period, the age-specific death rates of the total population will vary, the pattern of which will be determined by the initial age-sex composition as well as the fertility and the mortality rates. That is to say, the simultaneous continuation of the sex-age- specific rates and the age-specific rates for the total population may not be possible for any length of time unless the population are already stationary to start with.

In the same way, if a life table corresponding to a given schedule of mortality rates is needed then the construction of such a table cannot be dictated by any constraint extraneous to the data set. An attempt has been made in this paper to show why some of these constraints have little or no merit by removing their deceptive cloaks of consistency.

References

Lutz, Wolfgang and Sergei Scherbov, 1992, Sensitivity of aggregate period life expectancy to different averaging procedure. *Population Bulletin of the United Nations*, No. 33, pp. 32-46. United

Nations, 1989, *World Population Prospects*, 1988. Sales No. E.88.XIII.7. _____, 1991,

World Population Prospects, 1990. Sales No. E91.XHI.4.