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A General Index of Aging†

Introduction

AN index of aging of a population is a measure of the extent of its oldness. The construction of an aging index requires the undertaking of two distinct exercises: (i) the *identification* of the set of old persons in the population, and (ii) *aggregation* of the information available on the ages of the old persons into an indicator that will quantify the extent of oldness. The identification problem requires the specification of a cut-off point for old age. All persons exceeding this age are classified as old. We will refer to this cut-off point as old-age line. Thus, old-age line is a line of demarcation that separates the old persons from the set of non-old persons.

The most commonly used index of aging is the head-count ratio, the proportion of old persons in the population. But this index docs not indicate how old the old are. Moreover, it does not satisfy the properties of continuity and monotonicity suggested by Basu and Basu (1987). They introduced some new measures that fulfill these properties. Kulkarni (1988) noted that Basu and Basu (BB, for short) did not specify anything regarding the upper bound of an aging index. He made some suggestions along this line and modified the BB measures accordingly.

The purpose of this paper is to suggest some additional important properties for an aging index. We also propose a new general index that satisfies all these properties along with continuity and monotonicity. It contains the BB indices as special cases. Modification of the general index along the Kulkarni line is also suggested.

The paper is organized as follows. The next section discusses the properties for an aging index. The general index is introduced in Section 3. Section 4 presents a numerical illustration of the general index using age data of Indian population for the year 1981 and Section 5 makes some concluding remarks.

2. Properties for an Aging Index

Let $y \geq 0$ be the age of person *i* in an w-person society, where $n \geq 1$ is arbitrary. We write *y* for the age vector ($y_1, y_2, ...,y_n$ The cut-off point for old age is *g*. The *i*th person is old or non-old according as $y_i > g$ or $y_i \leq g$. For any age vector *y*, let g(y) be the set of all

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old persons. An aging index is a real valued function *A* which, given an old-age line *g*, associates to each age vector *y*, a value A *(g, y)* indicating its level of oldness.

We now suggest some postulates for an arbitrary index *A*. It is assumed that *g* is given exogeneouslv[1].

Focus (*F*): For any two age vectors $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$, if $g(x) = g(y)$ and $x_i = y_i$ for all $i \in g(x)$, then $A(g, x) = A(g, y)$.

Continuity (*C*): $A(g, y)$ is continuous in its arguments.

Monotonicity (*M*): $A(g, y)$ is increasing in y_i's, where $i \in g(y)$.

Symmetry (*S*): $A(g, y)$ is symmetric in $y_1, y_2, \ldots, y_n$.

Normalization (*N*): $A(g, y) = 0$ if the set $g(y)$ is empty.

Principle of population (*P*): Let z be an m-fold replication of the age vector y, that is, $z = (\underbrace{y_1, y_2, \ldots, y}_{m \text{ times}})$, then $A(g, z) = A(g, y)$.

Additive Decomposability (*D*): Suppose that the population is broken down into some groups (say k), defined along ethnic or geographical or other lines. Let y^i be the age vector of group i and n_i be its population size, so that $y = (y^1, y^2, \ldots, y^k)$ and $\sum_{i=1}^{k} n_i = n$.

Then
$$A(g, y) = \sum_{i=1}^{k} \frac{n_i}{n} A(g, y^i) \tag{1}$$

Axiom *F* specifies that once the old-age line is set, we disregard information on the age of the old persons in constructing an aging index. That is, an aging index is independent of the ages of the non-old persons, (The term 'focus' is borrowed from poverty measurement literature, see Sen (1981) and Chakravarty (1990). The next two axioms, namely continuity and monotonicity, were suggested by BB. Continuity means that small changes in one or more arguments of *A* will generate small changes in the functional value of *A*. Thus, small observational errors in ages will not make the index over-sensitive. The monotonicity axiom says that the index is increasing in the ages of the old persons. According to condition 5, the aging index is invariant under any permutation of its arguments. That is, the individuals are not distinguished by anything other than their ages. An implication of axiom 5 is that the index can directly be defined on ordered age vectors. Normalization principle says that the aging index takes on the minimum value (zero) whenever there is no old person in the society. It may be noted that an aging index satisfying axioms *N* and *M* will be positive if there is at least one old person in the society. While axiom *N* is concerned with the lower bound of A, according to Kulkami it should achieve its upper bound (one) when all the old persons reach the maximum possible age *m*. He also notes that for some indices (his modifications of two of the BB indices) this bound is obtained when all the persons in the society reach the age *m*. (For illustrative purpose, Kulkami assumes that $m = 100$ years.)

1. To illustrate their measures numerically, Basu and Basu (1987) and Kulkami (1988) assumed that g = 65 years.

Quite often we may wish to compare the oldness of two societies (say, 1 and 2) with population sizes, T_1 and T_2 respectively, where T_1 is not necessarily equal to T_2. But for cross society comparison of oldness it is desirable that the population sizes in the societies be the same. Now, if the population of state 1 (2) is replicated T_2 (T_1)times, then the total population of the two replicated states becomes the same (namely, $T_1 T_2$). Evidently, given a g, we can then compare the aging levels of these replicated states. If an aging index satisfies axiom P, then its functional value for a society and its replicated form will be the same. This means that with the help of an aging index meeting axiom P we can compare oldness of different populations. Similarly, given a g, axiom P enables us to make inter-temporal comparisons of oldness of the same population. The next property says that if the population is partitioned into some groups with respect to some socio-economic or geographic or some other characteristic, then the overall aging is a weighted average of group aging levels, where the weights for different groups are their population shares. The quantity $q_i = (n_j/n) A(g, y^j)$ is the contribution of group y to overall oldness, while $I00q_i/A(g, y)$ is this group's percentage contribution. Increase in oldness in a given group (the rest remaining fixed), will increase aggregate oldness. Thus, group oldness and aggregate oldness move in the same direction.

All of the above postulates were stated under the assumption that the old-age line is given exogenously. But sometimes this line may change. For instance, it may increase because of improvement in general health or climatic conditions in a society. In such a case oldness of the old persons will decrease. We formalize this argument as follows :

Monotonicity in old-age line (L,) : For a given age vector y, an increase in the value of the old-age line will decrease the aging index.

3. A General Index of Aging

We will assume for the rest of the paper that y_i's are non-increasingly ordered:

$y_1 \geq y_2 \geq, \ldots, \geq y_n$. It is also supposed that there are q old persons in the society. That is $y_i > g$ for $i = 1,2, \ldots, q$ and $y_i \leq g$ for $i = q + 1, \ldots, n$. Note that under these assumptions $g(y) = [1, 2, \ldots, q]$.

Let 0_i be proportionate gap between y_i and g, that is,

$$0_i = Yi / g-1 \tag{2}$$

Clearly, 0_1 is positive if person i is old and non-positive otherwise. 0_i can be regarded as a measure of oldness of person /. As a general index of aging, we consider

Clearly, 0, is positive if person i is old and non-positive otherwise. 0, can be regarded as a measure of oldness of person i. As a general index of aging, we consider

$$I_r = \frac{1}{n} \sum_{i=1}^{q} O_i^r$$

$$= \frac{q}{n} \left[\frac{1}{q} \sum_{i=1}^{q} \left(\frac{y_i}{g} - 1 \right)^r \right]$$

where $r \geq 0$ is a constant. The term q/n in (3) is the head-count ratio H and the term within third brackets, which measures how old the old are, is the average of the rth power of excess over g of the old population normalized by g^r.

The measure I_0 is simply the head-count ratio H, while I_1 equals $(q/n)\,(1/q \sum_{i=1}^{q} (y_i - g))$, the product of H and the mean excess over g of the old population as a fraction of g. I_1 is one of the BB measures. Another BB measure that drops out as a special case of I_r is I_2, the product of H and the mean squared excess of the old over g normalized by g^2. The third BB measure is obtained by dividing I_1 by H, which means that this measure and I_1 give us the same information regarding oldness of the old.

The parameter r in (3) reflects different perceptions of oldness. A larger r gives greater emphasis to the oldness of the older persons. As $r \to \infty$, I_r approaches oldness of the oldest person. It is easy to see that I_r meets F, N, S, P and D for all permissible values of r. It meets C,M and L for all r>0.

If we wish to modify I_r along the line suggested by Kulkarni, then the modified index will be

$$T_r(g, y) = \frac{1}{n} \sum_{i=1}^{q} \left(\frac{y_i - g}{m - g} \right)^r \tag{4}$$

For any $r > 0$, T_r takes on the value unity when all the w-persons reach the age m. Two of the modified BB measures, as considered by Kulkarni, are obtained as special cases of T_r if r = 1 and 2.

4. Numerical Illustration

The purpose of this section is to provide an empirical illustration of the general index I_r for some alternative values of r using age data of Indian population for the year 1981 (data source : Social and Cultural Tables, Census of India 1981, Part IV-A). The values of r we consider here are 0,1,2 and 3. Since I_0 is the head-count ratio, I_1 and I_2 are two of the BB measures, we are estimating three existing indices as well as a new index T_3. The analysis will enable us to compare the assessment of the existing indices I_0, I_1 and I_2 with that of the new index I_3.

We subdivide the entire population by region (rural-urban), sex and states. Overall aging indices as well as groupwise indices for each partitioning of the population were computed to create aging profiles.

The numerical results are given in Tables 1,2 and 3, the formats of which are identical. The first column gives the decomposition of the population with respect to a characteristic, whereas the second column presents the proportions of the total population in different subgroups. The third column gives the numerical values of the head-count ratio Io. In columns 4,5 and 6 we present the numerical values of I_r for r = 1,2 and 3 respectively. The subgroup aging levels are now weighted by the corresponding population shares to determine the contributions of different subgroups to total aging which are given as percentages of total aging in columns 7-10. It may be noted here that following BB we have considered 65 years as the cut-off point for old age.

TABLE 1 : MEASURES OF AGING FOR THE POPULATION IN INDIA IN 1981, BY REGION (RURAL-URBAN)

Region	*Proportion of all India population*	Io	I_1	I_2	I_3	*Percentage contributions to total aging based on*			
						Io	I_1	I_2	I_3
(1)	*(2)*	*(3)*	W	*(5)*	*(6)*	*(7)*	(8)	(9)	*(10)*
Rural	.763	.040	.312	4.200	77.345	80.331	80.129	79.823	79.564
Urban	.237	.031	.249	3.417	63.954	19.669	19.871	20.177	20.436
All India	1.000	.038	.297	4.014	74.171	100.000	100.000	100.000	100.000

TABLE 2 : MEASURES OF AGING FOR THE POPULATION IN INDIA IN 1981, BY SEX

Set	*Proportion of all India population*	Io	I_1	I_2	I_3	*Percentage contributions to total aging based on*			
						Io	I_1	I_2	I_3
(1)	*(2)*	*(3)*	(4)	(5)	(6)	(7)	(8)	*(9)*	*(10)*
Male	.517	.037	.288	3.859	70.355	50.548	50.260	49.692	49.037
Female	.483	.039	.305	4.181	78.254	49.452	49.740	50.308	50.963
All India	1.000	.038	.297	4.014	74.171	100.000	100.000	100.000	100.000

Some common features emerge from Tables 1-3. As argued earlier, we note that for a given population the index value increases as r increases. For instance, in the case of partitioning of the entire population by sex, the index for female population increases as r rises from 0 to 4. Next, as expected, for any subdivision of the total population by any characteristic, the percentage contributions made by different subgroups for alternative values of the parameter r do not differ much. However, the contribution made by a particular subgroup to total aging need not be monotonic in r. This seems intuitively reasonable. Since for calculating a particular subgroup's contribution we divide the subgroup index by the overall index, the resulting expression becomes a ratio of two polynomials of the same degree. Consequently, monotonicity of the subgroup contribution in r is not guaranteed.

From Table 1 we note that almost 80% contribution to total aging comes from the rural population. This happens because of high concentration of the Indian population (and hence expectedly of old persons) in the rural area. However, Table 2 shows that the contribution of the male population as a group does not differ much from that made by the female population. Further, I_r ranks the subgroups considered in Table (2) in the same way for all values of r.

TABLE 3 : MEASURES OF AGING FOR THE POPULATION IN INDIA IN 1981, BY STATES

State	*Proportion of All India population*	Io	I_1	I_2	I_3	*Percentage contributions to total aging based on*			
						Io	I_1	I_2	I_3
(1)	*(2)*	*(3)*	(4)	(5)	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*
Andhra Pradesh	.0759	.036	.281	3.568	60.595	7.739	7.628	7.155	6.576
Bihar	.0992	.037	.284	3.793	70.004	10.346	10.062	9.831	9.519
Gujarat	.0483	.035	.267	3.580	66.628	4.745	4.611	4.569	4.602
Haryana	.0183	.037	.293	3.933	71.928	1.917	1.917	1.903	1.784
Himachal Pradesh	.0060	.047	.406	5.954	116.794	.800	.880	.954	1.013
Jammu and Kashmir	.0085	.033	.300	4.607	95.282	.793	.911	1.033	1.156
Kamataka	.0526	.037	.325	4.681	90.121	5.560	6.114	6.410	6.682
Kerala	.0361	.048	.369	4.803	81.743	4.922	4.765	4.578	4.117
Madhya Pradesh	.1307	.021	.171	2.357	44.227	7.859	8.003	8.137	8.162
Maharashstra	.0891	.038	.290	3.990	76.271	9.457	9.275	9.631	9.604
Manipur	.0200	.037	.305	4.229	77.431	.205	.219	.225	.223
Meghalaya	.0019	.025	.207	2.872	53.402	.136	.140	.150	.145
Nagaland	.0011	.039	.396	6.780	150.008	.120	.155	.187	.236
Orissa	.0374	.038	.289	3.817	67.997	3.943	3.860	3.669	3.634
Punjab	.0238	.049	.437	6.564	131.354	3.270	3.717	4.027	4.370
Rajasthan	.0486	.033	.246	3.045	52.548	4.527	4.263	4.000	3.649
Sikkim	.0004	.025	.191	2.456	42.471	.032	.031	.029	.027
Tamilnadu	.0687	.037	.270	3.401	57.994	7.161	6.571	6.164	6.589
Tripura	.0029	.045	.442	6.961	141.910	.372	.460	.535	.590
Uuar Pradesh	.1572	.040	.324	4.520	86.185	17.800	18.201	18.624	19.093

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Stale	*Proportion of All India*	Io	I_1	I_2	I_3	I_0	I_1	I_2	I_3
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*
West Bengal	.0774	.033	256	3.451	63.908	7.173	7.078	7.043	7.069
Andaman	.0003	.017	.144	2.160	43.965	.013	.014	.015	.017
Arunachal Pradesh	.0008	.026	.219	9.646	61.572	.066	.070	.064	.049
Chandigarh	.0007	.025	.209	3.045	59.458	.044	.048	.052	.054
Dadra and Nagar Haveli	.0002	.021	.133	1.564	26.073	.009	.007	.006	.006
Delhi	.0088	.026	.215	3.055	58.697	.658	.679	.702	.740
Goa	.0015	.040	.299	3.812	66.525	.175	.165	.155	.147
Lakshadweep	.0001	.028	.213	2.922	56.153	.005	.004	.004	.005
Mizoram	.0007	.029	.253	3.685	70.103	.057	.063	.068	.070
Pondicheny	.0008	.040	.291	3.585	58.928	.096	.089	.080	.072
All India	1.0000	.038	.297	4.014	74.171	100.000	100.000	100.000	100.000

From Table 3, we note that in the case of statewise decomposition Uttar Pradesh contributes maximum to total aging. In contrast, the minimum contribution comes from Lakshadweep. Again concentration of population may be one of the reasons behind this. In fact, using head-count ratio we note that the Union Teritories of Andaman, Chandigarh, Dadra and Nagar Haveli, Goa, Lakshadweep and Pondicherry jointly constitute only .345% of total aging. This figure is much less than the individual contributions of many states like Bihar, Orissa, West Bengal, Gujarat, Madhya Pradesh, Maharashtra, Kerala, Tamilnadu, Kamataka, Andhra Pradesh, Rajasthan, Punjab and, of course, Uttar Pradesh. The ranking of states by I_0 agrees quite well with that based on I_1, I_2 or I_3. Spearman's rank correlation between any two indices turns out to be quite high.

Since the two general classes I_r and T_r are related ($I_r g^r/(m - g)^r = T_r$), we did not estimate T_r separately. With given values of *m* and *g*, for any decomposition of the population, the percentage contributions made by different subgroups of the decomposed age distribution will be the same under I_r and T_r for all values of *r*. Furthermore, for any *r*, the ranking of the subgroups produced by I_r will coincide with that generated by T_r.

5. Conclusion

Basu and Basu (1987) noted that the head-count ratio, the proportion of population exceeding a pre-designated cut-off point for old age, is not monotonic and continuous in the ages of the old. They therefore suggested some measure of aging that fulfils these criteria. Kulkarni (1988) proposed a modification of the Basu and Basu measures. In this paper we have suggested some new properties for an aging index and introduced a general index that meets these properties along with monotonicity and continuity. This general formula contains the Basu and Basu indices as special cases. Finally, we provide an empirical illustration of alternative indices using age data of Indian population.

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