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Graduation of Infant Deaths by Age

Introduction

BIRTH and Death registered through vital registration system in the developing countries suffer from omissions and under count. Partly with idea to overcome this breakdown supposed by defective data and to obtain reliable estimates of birth and death rates, India introduced Sample Registration System (SRS) in 1960. Although the reliability and quality of data on births and deaths recorded in the civil registration system have considerably improved overtime, they still suffer from considerable degree of errors of omission and mismatching. It is observed in retrospective surveys that events are misreported partly due to ignorance and partly due to tendency of digit preference of the respondents. Thus, the data on deaths collected through vital registration or Sample Registration Systems or retrospective surveys suffer from one defect or other as mentioned above. Partly to overcome this problem attempts have been made to develop and fit suitable models to data on age distribution of deaths. For death rates (not deaths), so as to remove the variations from one age to other age and smooth the data, which reflect the underlying age pattern of the mortality (i.e., mortality rates).

In this paper an attempt has been made to develop a model, an adoption of an earlier one, which will take into account the nature of declining tempo of deaths by age during the first year of life. The model is used to give a functional shape to the phenomenon of infant deaths distribution at different ages.

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Justification of Study

It is well known that among the many available indicators of socio-economic development, Infant Mortality Rate (IMR) is used as a sensitive and powerful index of development. Reduction in IMR is also known to reduce the fertility as probability of survivorship of children increases. It is also used to compare health situations among different communities.

Infant deaths occur due to endogenous and exogenous causes. Endogenous causes are biological whereas exogenous causes are environmental. As it has been observed in developing countries that number of infant deaths registered is subject to error of misreporting, which also distorts the distribution of deaths by age during infancy.

Hence a method for redistribution of infant deaths by months during the first year is important to get a clear picture on the number of deaths at various age points under one year.

Objectives

The specific objectives of the paper are:

- (i) to develop a methodology of proposed model for distribution of deaths by age within infancy,
- (ii) to estimate the extent of errors observed due to misreporting of infant deaths in Kamataka.

Sources of Data

The data used in this paper have been taken from following sources:

- (1) For Sweden, "foetal, infant and early childhood mortality", Publication of United Nations, New York, 1954.
- (2) For United States, from "Vital Statistics rates" in U.S. (1940-60).
- (3) For Kamataka, "NFHS Kamataka 1992-93 report".
- (4) For, Tamil Nadu "NFHS report of Tamil Nadu 1992".

Review of Literature

Some of the studies relating to the age pattern of mortality have developed models of mortality, which are mathematical expressions for graduation of age pattern of mortality. They are mostly suitable to graduate mortality rates by age after age 30.

Theile (1972) has proposed a seven-parameter model to capture age pattern of mortality for the whole age range. He has considered three different factors, exponential in nature, representing young, adult and old age mortality pattern. Later Heligman and Pollard (1980) have developed a eight-parameter model and tested the same on Australian data at different points of time. They also gave three different factors like Theile. The model of Heligman and Pollard serves to give better fit.

No suitable model has been developed specifically to graduate infant deaths within the first year of life. The model developed recently by Mukherjee and Islam (1983) will be adopted and used in the present study.

Finite Range Model

This model was proposed by Mukherjee and Islam (1983) for Reliability Analysis. Here the model given as probability density function (pdf), which gives the probabilities of deaths distributed among total infant deaths by age, is as follows.

$$f(x) = \frac{P}{x} \left(\frac{x}{\theta}\right)^p \tag{1}$$

$$0 < x < \theta; \theta > 0 \text{ and } 0 < p < 1$$

Here θ and p are the parameters of the model. θ is scale parameter and p determines the shape.

Equation (2) below gives cumulative distribution function (cdf). The cdf gives the cumulative proportion of deaths up to a desired age point 'x'.

$$F(X < x) = \frac{(X)^p}{\theta} \tag{2}$$

$$0 < x < \theta; \theta > 0 \text{ and } 0 < p < 1$$

Another important character of the pdf (1) is the Moment Generating Function (MGF) which is given by this functional relation.

$$M_X(t) = \sum_{s=0}^{p-\infty} \frac{\theta^{s+p} t^s}{s!} \tag{3}$$

In order to get r th moment about zero the identity used is as follows

$$\mu_r = \left[\frac{d^r}{dt^r} M_x(t) \right]_{t=0} \quad (4)$$

By using Eqns. (3) and (4) the expressions of mean and variance of the variate X are derived as

$$\text{Mean} = \left(\frac{p}{p+1} \right) \theta \quad (5)$$

$$V(x) = \left[\frac{p}{(p+1)(p+2)^2} \right] \theta^2 \quad (6)$$

and median of distn. (1) is computed as

$$\text{Mean} = \left[\left(\frac{1}{2} \right)^{\frac{1}{p}} \right] \theta \quad (7)$$

Likewise percentile and decile values can also be derived easily. If we look upon the graphical shape of the pdf (1), it is of mirror-J shaped curve decreasing with x . The graph for the different values of the p is shown in Figure A.

Methodology

Certain assumptions underlie the application of the above model to data in infant deaths by age have been developed. They are:

Assumptions

It is necessary to fulfil three assumptions given below for fitting the curve.

- (1) Effect of seasonal variations on deaths at different ages, if any, is independent of age during infancy (or evenly distributed).
- (2) No misreporting of the neonatal deaths or deaths of infants with one month of life.

- (3) There may be shortage of frequencies (due to misreporting) for certain preferred ages as 3rd, 6th month etc. but there is no over reporting or under reporting of total deaths as far as the first year of life is concerned; i.e. total is correctly reported.

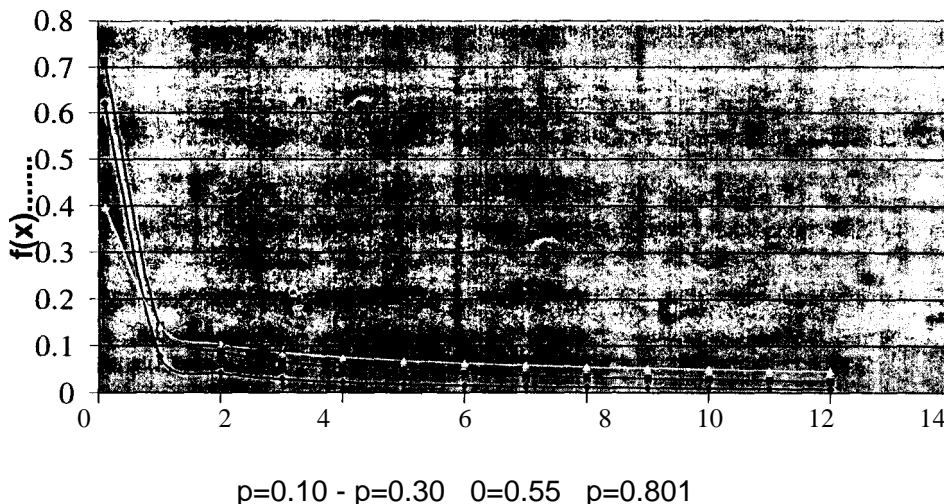


Fig. A. Shape of pdf (1) The

methodology involves the following steps:

Step (I)

In order to fit the proposed curve from data on total and neonatal deaths, estimation of the two parameters is necessary. The method of iteration was used and the fit was tested by minimum Chi-square.

If D and N be the respective total number of infant and neonatal deaths reported in a geographical area during the reference period (neonatal deaths refer to the deaths during first month of life). If R denotes the proportion of neonatal deaths among the total infant deaths.

$$\text{or } R = N/D \tag{8}$$

Now fixing θ as 12 months, using the data available as "reported age of death in months".

By Eqn. (2) we can estimate the total proportion of deaths under the age points

$$F(X < x) = \left(\frac{x}{\theta}\right)^p \quad (9)$$

$$0 < x < 12$$

in (9) if we put $x = 1$, then we will get the total proportion of deaths up to the first month of age to total Infant Deaths

$$F(X < x) = \left(\frac{1}{12}\right)^p \quad (10)$$

by Eqns. (8) and (10)

$$R = \left(\frac{1}{12}\right)^p \quad (11)$$

By solving (11) we get the estimated value of p as

$$\hat{p} = \frac{\text{Log}(R)}{\text{Log}(1/12)} \quad (12)$$

Step (II)

The above estimated value of p i.e., \hat{p} as the first approximation can be used in eqns. (1) and (2) to have the proportion of death for given data. So (1) will be as

$$f(x) = \left(\frac{\hat{p}}{x}\right) \left(\frac{x}{\theta}\right)^{\hat{p}} \quad (13)$$

$$0 < x < 12$$

and cdf will be given by (2) as

$$F(X < x) = \left(\frac{x}{12}\right)^{\hat{p}} \quad (14)$$

$$0 < x < 12$$

by putting $x = 1, 2 \dots 12$ in the Eqn. (14) we will get the cumulative proportion of death up to age of first month, second month, etc., up to twelfth month.

Step (III)

To get the proportion of deaths during a particular month of age we have to subtract the successive values as proportion of deaths during month x

$$\begin{aligned} &= \text{Proportion of deaths up to } x + 1' - \text{Proportion of deaths up to age 'x'} \\ &= F(x < x + 1) - F(X < x); \quad x > 1 \end{aligned} \tag{15}$$

and

$$\text{proportion of deaths during first month} = F(X < 1) \tag{16}$$

Then number of deaths in first month and onward can be estimated by multiplying eqn. (16) and eqn. (15) by D , so if we denote

$$\begin{aligned} d_1 &= D * F(X < 1) \\ dx &= D * F(X < x) \end{aligned} \tag{17}$$

Goodness of Fit

By eqn. (17) we derive the expected deaths E_i and observed deaths O_i are already available to us. We can use the chi-square test of goodness of fit to test whether the model fits the observed data.

$$\chi^2 = \sum_{i=1}^{12} \frac{(O_i - E_i)^2}{E_i} \tag{18}$$

By (18) we can compute the test statistic (Chi-Sq.) and test it for significance for appropriate degrees of freedom.

Analysis and Findings

The validity of model is first testified by fitting it to Swedish data on infant deaths by age in months at three periods of time (i.e. 1915-24, 1925-34 and 1935-44). Data on age at deaths of infants in Sweden are considered to be highly reliable. The infant mortality rates of Sweden for the selected three periods were 65.1, 54.9 and 37.3, which are closer to the existing IMR of some Indian States. The fitting

TABLE 1: FITTING FOR SWEDEN
(Deaths per thousand live birth)

Age at death (in months)	Observed deaths	$F(X < x)$	Proportion of deaths (in months)	Expected deaths	Characteristics
1915-24					
0 - 1	26.3	0.394303	0.394303	26.30003	$R = 0.3943028$
1 - 2	7.5	0.511176	0.116873	7.795426	
2 - 3	5.8	0.595003	0.083827	5.591275	$p = 0.374515$
3 - 4	4.3	0.662691	0.067687	4.514744	
4 - 5	3.6	0.720452	0.057761	3.852688	$\chi^2_{\text{calculated}} = 0.109005$
5 - 6	3.2	0.771365	0.050912	3.395861	
6 - 7	3	0.817207	0.045843	3.057714	$\chi^2_{\text{diff \& 5\%}} = 11.07$
7 - 8	2.9	0.859115	0.041907	2.795216	
8 - 9	2.8	0.89786	0.038745	2.5843	
9 - 10	2.6	0.933997	0.036137	2.410339	
10 - 11	2.4	0.967938	0.033941	2.263883	
11 - 12	2.3	1	0.032062	2.138524	
0 - 11	66.7		1	66.7	
1925-34					
0 - 1	26.1	0.466072	0.466072	26.10002	$R = 0.4660714$
1 - 2	5.4	0.576681	0.110609	6.19411	
2 - 3	4.5	0.653182	0.076502	4.284086	$p = 0.307221$
3 - 4	3.4	0.71354	0.060358	3.380022	
4 - 5	2.8	0.764172	0.050632	2.835392	$\chi^2_{\text{calculated}} = 0.143289$
5 - 6	2.5	0.808197	0.044025	2.465403	
6 - 7	2.1	0.847393	0.039196	2.194957	
7 - 8	2.1	0.882879	0.035486	1.987219	$\chi^2_{\text{diff \& 5\%}} = 9.488$
8 - 9	2	0.915411	0.032532	1.821814	
9 - 10	1.9	0.945527	0.030116	1.686478	
10 - 11	1.7	0.973622	0.028096	1.573352	
11 - 12	1.5	1	0.026378	1.477145	
0 - 11	56		1	56	
1935-44					
0 - 1	22	0.580525	0.580525	22.00191	$R = 0.5804749$
1 - 2	2.9	0.675619	0.095093	3.604035	
2 - 3	2.4	0.738311	0.062692	2.376031	$p = 0.218885$
3 - 4	1.9	0.786289	0.047978	1.818364	
4 - 5	1.5	0.82564	0.039351	1.491419	$\chi^2_{\text{calculated}} = 0.124232$
5 - 6	1.4	0.85925	0.03361	1.273819	
6 - 7	1.3	0.888732	0.029482	1.11737	
7 - 8	1.1	0.915087	0.026355	0.998852	$\chi^2_{\text{diff \& 5\%}} = 3.84$
8 - 9	1	0.938982	0.023895	0.905608	
9 - 10	0.9	0.960884	0.021903	0.830113	
10 - 11	0.8	1	0.018862	0.71488	
0 - 11	37.9		1	37.9	

Source: "Foetal, Infant and Child Mortality Vol. I the Statistics (Population Studies No.13)", U.N. Social Affairs, New York, (1954: 33).

is also done for the 1960 United States Vital Registration Data of Infant Deaths by age in month, to show the extent of fit for another developed country with reliable data.

The model fitted for Swedish data at three time periods by first estimating the parameters and then estimating the expected deaths is explained above. Table 1 shows the observed and expected distribution of deaths for each month. The fit is very good as the respective chi-sq. values are within the range given in the table.

The fit for United States is given in the Table 2, and the expected values according to the model are quite close to observed values. The calculated value of chi-sq. is 16.11 where the tabulated value of chi-sq. (10 df and 5 percent level of significance) is 18.307. So the fitted distribution is fairly good, though not as good as for Sweden.

TABLE 2: **FITTING FOR UNITED STATES 1960** (Deaths per hundred thousand live births: Vital Statistics)

<i>Age at Death</i> (in months)	<i>Observed deaths</i>	<i>F(X < x)</i>	<i>Proportion of deaths</i> (in months)	<i>Expected deaths</i>	<i>Characteristics</i>
0 - 1	1872.8	0.719202	0.719202	1872.802	$R = 0.719201$
1 - 2	170.8	0.788463	0.069261	180.3556	$p = 0.132646$
2 - 3	132.9	0.83203	0.043567	113.4491	
3 - 4	100.7	0.864394	0.032364	84.27519	$\chi^2_{\text{calculated}} = 16.11$
4 - 5	77.2	0.890362	0.025968	67.61989	
5 - 6	57.4	0.912157	0.021795	56.75469	
6 - 7	47.2	0.931	0.018843	49.06794	$\chi^2_{10 \text{ df} \& 5\%} = 18.307$
7 - 8	39.2	0.947637	0.016637	43.32317	
8 - 9	33.2	0.962559	0.014922	38.85591	
9 - 10	26.7	0.976106	0.013547	35.27595	
10 - 11	23	0.988525	0.012419	32.33847	
11 - 12	22.9	1	0.011475	29.88184	
0 - 11	2604		1	2604	

Source: Vital Statistics Rates in U.S. 1940-60 by Robert D. Grove and Alice M. Hetzel, Washington, D.C. (1968: 210-211).

For Indian data the model is tested for two states Tamil Nadu and Karnataka. The data are taken from NFHS Reports for both the states. Table 3 gives the distribution of observed and expected number of deaths by age for Tamil Nadu. The calculated value of chi-sq. is 14.49 and tabulated value of chi-sq. (for 8 degrees of freedom and 5 percent level of significance) is 15.507. Hence it is clear that fitted curve for Tamil Nadu is fairly good.

TABLE 3: FITTING FOR TAMIL NADU (NPHS 1992-93)

<i>Age at Death (In months)</i>	<i>Observed deaths</i>	<i>F (X < x)</i>	<i>Proportion of deaths (in months)</i>	<i>Expected deaths</i>	<i>Characteristics</i>
0 - 1	199	0.670033	0.670033	198.9999	$R = 0.670034$
1 - 2	21	0.749214	0.07918	23.51651	
2 - 3	7	0.7998	0.050587	15.02432	$p = 0.161144$
3 - 4	15	0.837751	0.03795	11.27121	
4 - 5	6	0.868423	0.030672	9.109642	$\chi^2_{\text{calculated}} = 14.49$
5 - 6	5	0.894316	0.025893	7.690158	
6 - 7	12	0.916809	0.022493	6.680548	
7 - 8	7	0.936751	0.019941	5.922619	
8 - 9	8	0.9547	0.017949	5.330958	$\chi^2_{8 \text{ df} \& 5\%} = 15.507$
9 - 10	4	0.971047	0.016347	4.855202	
10 • - 11	7	0.986076	0.015029	4.463645	
11 • - 12	6	1	0.013924	4.135285	
0 - 11	297		1	297	

Source: National Family Health Survey Tamil Nadu 1992-93.

TABLE 4: FITTING FOR KARNATAKA (NFHS 1992)

<i>Age at death (in months)</i>	<i>Observed deaths</i>	<i>F(X < x)</i>	<i>Proportion of deaths (in months)</i>	<i>Expected deaths</i>	<i>Characteristics</i>
0 - 1	303	0.685521	0.685521	303.0001	$R = 0.68552$
1 - 2	32	0.761661	0.07614	33.65383	
n 3	21	0.810062	0.048401	21.39337	$p = 0.151948$
3 - 4	22	0.846257	0.036195	15.99834	
4 - 5	6	0.875443	0.029185	12.89993	$\chi^2_{\text{calculated}} = 14.96$
5 - 6	9	0.900034	0.024592	10.86957	
6 - 7	13	0.921365	0.02133	9.427965	
7 - 8	9	0.94025	0.018885	8.34729	$\chi^2_{9 \text{ df} \& 5\%} = 16.919$
8 - 9	5	0.957229	0.016979	7.504735	
9 - 10	13	0.972677	0.015448	6.827976	
10 - 11	4	0.986866	0.014189	6.271533	
11 - 12	5	1	0.013134	5.805311	
0 - 11	442		1	442	

Source: National Family Health Survey Karnaiaka 1992 (1995: 222)

Table 4 gives the distribution of observed and expected deaths for Karnataka from first to twelfth month of age at death. The calculated chi-sq. value found to be 14.96 which is less than the tabulated value of chi-sq. (for 9 degrees of freedom and 5 percent level of significance) 16.919. So the fit is also good for Karnataka State. From the table it is quite clear that there is a heaping for the seventh and tenth month. So by looking at both columns we can say that expected deaths are a reasonable redistribution of observed deaths.

Table 5 gives an estimate of the extent of misreporting of infant deaths (for the 5-9 years preceding survey). The procedure used here is similar to the one

TABLE 5: ESTIMATION OF EXTENT OF MIS-REPORTING, Karnataka, NFHS 1992-93 (5-9 Years preceding survey)

<i>Age at death</i> (in months)	<i>Observed deaths</i>	<i>F(X < x)</i>	<i>Proportion of deaths</i> (in months)	<i>Expected deaths</i>	<i>Characteristics</i>
0 - 1	174	0.564934	0.564934	173.9998	$R = 0.564935$
1 - 2	22	0.64337	0.078435	24.15812	
2 - 3	13	0.694207	0.050838	15.65797	$p = 0.187565$
3 - 4	14	0.732695	0.038488	11.85425	
4 - 5	5	0.764012	0.031317	9.645624	$\chi^2_{\text{calculated}} = 20.08$
5 - 6	8	0.790591	0.026579	8.186302	
6 - 7	7	0.813783	0.023192	7.14321	$\chi^2_{11 \text{ df } \& 25\%} = 21.920$
7 - 8	5	0.834423	0.020639	6.356889	
8 - 9	5	0.853062	0.018639	5.740865	Reported Infant
9 - 10	11	0.870088	0.017026	5.243957	Deaths = 267
10 - 11	2	0.885782	0.015694	4.833842	
11 - 12	1	0.900357	0.014575	4.489046	
12 - 13	20	0.913976	0.013619	4.19472	Calculated Infant
13 - 14	2	0.926769	0.012793	3.940252	Deaths = 277.3098
14 - 15	1	0.93884	0.012071	3.717845	
15 - 16	2	0.950274	0.011434	3.521634	Deaths Under-
16 - 17	2	0.961141	0.010867	3.347124	reported = 10.3098
17 - 18	12	0.971501	0.01036	3.190802	
18 - 19	0	0.981403	0.009902	3.049886	Percent under
19 - 20	1	0.99089	0.009487	2.922142	reporting = 3.72%
20 - 21	1	1	0.00911	2.80575	
0 - 21	308		1	308	

Source: National Family Health Survey Karnataka 1992 (1995: 222)

TABLE 6: ESTIMATION OF EXTENT OF MIS-REPORTING
Karnataka, NFHS 1992-93
(0-10 Years preceding survey)

Age at death (in months)	Observed deaths	$F(X < x)$	Proportion of deaths (in months)	Expected deaths	Characteristics
0 - 1	303	0.610887	0.610887	303	$R = 0.610887$
1 - 2	32	0.680211	0.069324	34.38476	
2 - 3	21	0.724355	0.044144	21.89534	$p = 0.155077$
3 - 4	22	0.757402	0.033047	16.39143	
4 - 5	6	0.784071	0.026668	13.22745	$\chi^2_{\text{calculated}} = 95.63$
5 - 6	9	0.806556	0.022485	11.1526	
6 - 7	13	0.826069	0.019513	9.678547	$\chi^2_{13 \text{ df } \& 5\%} = 22.362$
7 - 8	9	0.843353	0.017284	8.573005	
8 - 9	5	0.858899	0.015546	7.710696	
9 - 10	13	0.873048	0.014149	7.017813	Reported Infant
10 - 11	4	0.886048	0.013	6.447923	Deaths = 442
11 - 12	5	0.898084	0.012037	5.970291	Calculated Infant
12 - 13	25	0.909302	0.011217	5.563732	Deaths = 445.4499
13 - 14	3	0.919812	0.01051	5.213146	
14 - 15	1	0.929706	0.009894	4.907471	
15 - 16	4	0.939058	0.009352	4.638407	Deaths Under-
16 - 17	2	0.947928	0.00887	4.399603	reported = 3.4499
17 - 18	0	0.956368	0.00844	4.186111	
18 - 19	14	0.96442	0.008052	3.994017	Percent under-
19 - 20	1	0.972122	0.007702	3.820184	reporting = 0.77%
20 - 21	2	0.979505	0.007383	3.662066	
21 - 22	1	0.986597	0.007092	3.517574	
22 - 23	0	0.993422	0.006825	3.384977	
23 - 24	1	1	0.006578	3.262829	
0 - 23	496		1	496	

Source: National Family Health Survey Karnataka 1992 (1995: 222).

explained in earlier section. Here the time duration is taken 21 months instead of 12 months. Accordingly parameter 'p' is estimated for Karnataka. The estimated

p is 0.187565 and expected deaths are computed for the months 1 to 21. The fitted curve is found to be a good fit as tested the chi-sq. values (for $\alpha = 2.5\%$). Hence curve can be extended for later ages. The sum of observed deaths for 0-11 months gives the corrected value of Infant Deaths. The difference between the observed and expected death is giving 10.31 deaths under reported (3.72 percent).

Table 6 gives the estimation of extent of misreporting (for the 0-10 years preceding survey). The procedure used here is same as explained for Table 5. Here the scale is taken 24 months instead of 12 months. Accordingly parameter ' θ ' is estimated for Kamataka. The estimated p is 0.155077 and expected deaths are computed for the month 1 to 24. For the age group 12-13 there is too much over reporting. So chi-sq. value is affected by that cell only. Here percent under reporting is 0.77%. This is quite obvious that by adding corresponding deaths in two periods i.e. 0-4 years and 5-9 years, some errors has been graduated automatically.

That's why in order to satisfy third assumption, 0-10 year reference period has been taken when computing Tables 1, 2, 3 and 4.

Further Applications

(i) To observe the centrality of death distribution we can use any of these two measures (i.e. median and mean) as single Index, e.g. medians for fitted distributions of Sweden, U.S., Tamil Nadu and Kamataka are 1.89 months, 1.94 days, 4.88 days and 3.76 days respectively. Similarly mean values of these distributions yielded to 3.27 months, 1.41 months, 1.67 months and 1.58 months respectively.

(ii) As the model fitting in different situation indicates the model can be extended up to the minima of age pattern of mortality.

(iii) If reliable data on neonatal and total infant deaths are known for any reference area, then by applying above model, deaths can be distributed month-wise.

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