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## Estimation of Fecundability of Migrated Couples : A Birth Interval Approach

### Introduction

BESIDES biological factors, a number of socio-economic and cultural factors affect human reproduction process. Migration, especially in developing countries plays an important role in changing the socio-economic, cultural and other conditions of the people involved with and hence affects the fertility (Goldstein *et al.* 1982; Mishra *et al.* 1983; Yadava and Yadava, 1990).

A number of probability distributions have been derived to study the fertility behaviour of migrated couples' based on the number of births during a specified period of time (Singh ;*et al.* 1981 and Yadava *et al.* 1992), but very few models have been developed to study the fertility behaviour of such couples based on data of birth intervals. In fact, birth interval models provide more sensitive indices for detecting current changes in fertility patterns of women who are still in reproductive ages than the indices based on the number of births (Chakraborty 1976 and George 1967).

Mishra *et al.* (1983) have derived a probability model for first birth interval (the duration of time from marriage to first birth) and estimated the fecundability of migrated couples. Models based on first birth interval give an idea about the fertility behaviour of couples in the beginning of their married life. However, this interval does not include post partum amenorrhea (ppa) which is an important factor for the study of fertility behaviour of a couple. This implies the importance of the study of the closed birth interval (an interval between two consecutive births). A fertility analysis based on closed birth intervals data is also useful to study the fertility of females after certain births.

This paper aims to derive a simple procedure for the estimation of fecundability of migrated couples based on the data of closed birth intervals. A probability distribution for the closed birth interval of migrated couples is also proposed. The theoretical distribution is illustrated with an observed set of real data collected from the rural areas.

### Procedure for the Estimation of Fecundability and its Application

Let  $p$  denote the fecundability defined as the probability of conception to a female in one menstrual cycle, if the female is exposed to the risk of conception. In this definition time is

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1. A migrated couple is defined as that whose male partner/was found to have migrated, leaving his wife in village and visits his home once or twice annually or in some cases even only once in two or three years.

discrete and its unit is a month. Let us consider a cohort of females with experience of sufficiently large marital duration having at least  $(i + 1)$ ,  $(i > 1)$  births and their husbands having started migration before the  $i$ th birth. Let in every year migrated husbands visit their wives randomly for a constant period, say for  $y$  months. Further, let us assume that all those females will be susceptible to risk of conception during the visit of their husbands in the second year who were not susceptible in the first year during the visit of their husbands after the  $i$ th birth.

Following Sharma (1984), the probability of conception during the stay of husband when he visits to his wife may be taken as

Let  $S$  be the proportion of females exposed to the risk of conception during the visit of their husbands in the first year after the  $i$ th birth. Since

$$\sum_{j=1}^y pq^{j-1}, \quad q = 1 - p \quad (1)$$

$$\sum_{j=1}^y pq^{j-1} = p^* \quad (\text{say})$$

is the probability of conception in a year, which is assumed to be constant for each and every year, so the proportion conceived during the first year after  $i$ th birth will be the product of proportion of females susceptible to risk of conception and probability of conception in a year, i.e.,  $Sp^*$ .

A conception may result either in a live birth or in a foetal loss. Let, out of conceived females,  $\alpha$  be the proportion of females responsible for foetal losses. So proportion of females whose conceptions result in foetal loss is  $\alpha Sp^*$ .

Thus proportion of conceived females whose  $(i + 1)$ th birth resulted in the live birth in the first year visit of their husbands is,

$$Sp^* - \alpha Sp^* = (1 - \alpha) Sp^* \quad (2)$$

Now the proportion of females susceptible to the risk of conception in the second year at the time of visit of their husbands after  $i$ th birth = proportion not susceptible in the first year + proportion of females not conceived (out of the susceptible females at the first year) during first year + proportion of females (out of conceived females in the first year) whose conception after  $i$ th birth results in foetal loss, i.e.,

$$= (1 - S) + (S - Sp^*) + \alpha Sp^*$$

$$= 1 - Sp^* + \alpha Sp^*$$

Thus, proportion conceived in the second year is

$$(1 - Sp^* + \alpha Sp^*) p^*$$

and the proportion resulted in foetal loss is

$$\alpha (1 - Sp^* + \alpha Sp^*) p^*$$

Hence, proportion resulted in  $(i + 1)$ th live birth is

$$(1 - \alpha) (1 - Sp^* + \alpha Sp^*) p^* \quad (3)$$

Now the proportion of females susceptible to the risk of conception in the third year at the time of visit of their husbands after  $i$ th birth = proportion not conceived (out of the susceptible females at second year) during 2nd year + proportion of females (out of conceived females in the 2nd year) whose conception resulted in foetal loss

$$= [(1 - Sp^* + \alpha Sp^*) - (1 - Sp^* + \alpha Sp^*) p^*] + \alpha [1 - Sp^* + \alpha Sp^*] p^*$$

$$= (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*)$$

Therefore, proportion conceived in third year

$$= (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*) p^*$$

and the proportion resulted in foetal loss in third year

$$= \alpha (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*) p^*$$

Hence, proportion resulted in  $(i + 1)$ th live birth in the third year after  $i$ th birth is

$$(1 - \alpha) (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*) p^* \tag{4}$$

Similarly, the proportion of females whose conception will result in  $(i + 1)$ th live birth after  $i$ th birth in the fourth year is

$$(1 - \alpha) (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*)^2 p^* \tag{5}$$

and in the fifth year is

$$(1 - \alpha) (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*)^3 p^*$$

and so on.

Now the theoretical closed birth intervals of the females conceived during the visit of their husbands in different years after the  $i$ th birth under the assumption that every conception results in live birth with a proportion of foetal losses in every conception will be as given below.

It may happen that some of the females may be conceived during the beginning of the year whereas some may be conceived in the middle and some may be conceived in the end of the year depending on the risk of susceptibility of the females and time of visits of their husbands. If we assume that number of females conceived during a year are uniformly distributed, then it may be considered that on average a female conceives in the middle of a year. Under this assumption the closed birth intervals for the conceptions in different years are given as:

<i>Year of conception after <math>i</math>th birth</i>	<i>Length of closed birth intervals</i>
I. First year	1.25 years (conception in 6th month + 9 months of gestation period)
II. Second year	2.25 years (conception in the second year's 6th month + 9 months of gestation)
.....	.....
.....	.....

In general if conception occurs in the  $X_i$  year after  $j$ th birth, then the length of closed birth interval is equal to

$(X_i + 0.25)$  years, for  $X_i = 1, 2, 3, \dots$

Thus the theoretical average length of closed birth interval (which is expected to be observed in the population) is given by

$$1.25 (1 - \alpha)Sp^* + 2.25 (1 - \alpha) (1 - Sp^* + \alpha Sp^*) p^* + 3.25 (1 - \alpha) (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*) p^* + 4.25 (1 - \alpha) (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*)^2 p^* + 5.25 (1 - \alpha) (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*)^3 p^* + \dots$$

After simplification it comes out to be

$$1.25 Sp^* (1 - \alpha) + \frac{(1 - Sp^* + \alpha Sp^*) (1 + 1.25p^* - 1.25\alpha p^*)}{p^* (1 - \alpha)} \quad (6)$$

The above procedure to estimate the level of fecundability for migrated couples is applied to an observed data set collected through a survey entitled "RDPG—A Sample Survey, 1978" from the rural areas of Uttar Pradesh (a brief description of the survey is given in the Appendix). The data for about 106 females whose husbands were found migrated since before first to at least till the second birth have been taken from the survey to illustrate this procedure and to estimate the level of fecundability of such couples. Thus, the application of the above procedure requires the knowledge of the values of  $a$ ,  $S$  and average length of the first closed birth interval. The average of the first closed birth interval is calculated from Table 2 and its value comes out to be 3.316 years. Although, to find an estimate of the proportion of foetal losses ( $a$ ) is not possible as we do not have information on this item in the survey. However, some Indian studies cited in Singh (1985) indicate that  $a$  varies from 0.05 to 0.10. Thus an estimate of fecundability can be obtained by equating the observed average length of the closed birth interval to its theoretical expression given by equation (6) provided the value of  $S$  is known. The estimate of fecundability for different values of  $S$ ,  $y$  and  $a$  are given in Table 1. The reason for taking the value of  $y = 2$  months is just like Singh *et al.* (1981) and that for  $y = 1.8$  months is based on the average duration of visit of husbands for females of age group less than 30 years respectively as in Yadava *et al.* (1992). This is only for the sake of comparison because females of age less than or equal to 30 years have much exposure time for at least two births.

Here, for  $y = 2$  months,  $p^* = p + p(1 - p)$

and for  $y = 1.8$  months,  $p^* = p + 0.8(1 - p)$

(from expression (1))

Thus, for example if  $S = 0.20$ , then  $p^*$  is equal to 0.4645 for  $a = 0.05$  and 0.4903 for  $a = 0.10$  and hence the value of  $p$  for  $y = 2$  months comes out to be 0.268 for  $a = 0.05$  and 0.297 for  $a = 0.10$ . For  $y = 1.8$  months the estimate of fecundability comes out to be 0.286 and 0.317 for  $a$  equal to 0.05 and 0.10 respectively. These estimates are reasonably higher than the estimates obtained by Kushwaha (1992). This difference may be mainly due to the ignorance of foetal losses by Kushwaha. It should, however, be mentioned that the level of

fecundability is not affected much by the variation in the value of  $S$ . Nevertheless the estimates of  $S$  and  $\alpha$  are required.

TABLE 1 : ESTIMATED VALUES OF FECUNDABILITY ( $p$ ) FOR DIFFERENT VALUES OF  $S$  AND  $y$

Different values of $S$	$p^*$		Estimates of fecundability ( $p$ )			
	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$		$\alpha = 0.10$	
			$y = 2$ months	$y = 1.8$ months	$y = 2$ months	$y = 1.8$ months
0.15	0.4750	0.5014	0.275	0.292	0.305	0.326
0.20	0.4645	0.4903	0.268	0.280	0.297	0.317
0.25	0.4545	0.4798	0.261	0.278	0.289	0.309
0.30	0.4449	0.4696	0.255	0.272	0.283	0.301
0.35	0.4357	0.4599	0.249	0.265	0.276	0.294
0.40	0.4269	0.4508	0.243	0.258	0.269	0.287

In the following section a probability model is proposed for first closed birth interval under the above discussed assumptions where the estimate of  $S$  is also obtained.

**A Probability Model for Closed Birth Interval of Migrated Couples**

Let  $X$  denote the length of closed birth interval. It may be considered as a random variable which may take values 1.25, 2.25, 3.25 . . . . Then its probability mass function, based on the assumptions of the previous section is given by

$$g(X) = \begin{cases} (1 - \alpha) Sp^* & \text{if } X = 1.25 \\ & X = 2.25 \\ (1 - \alpha) (1 - Sp^* + \alpha Sp^*) (1 - p^* + \alpha p^*) p^* & \text{if } X = 2.25, 3.25, \dots \end{cases}$$

where  $p^* = \sum_{j=1}^y p \cdot q^{j-1}$

*Estimation of the Parameters of the Model and its Application*

The model has three parameters ,  $\alpha$ ,  $S$  and  $p$ . To estimate the parameters of the model, the method of moments technique is applied and accordingly the first and second theoretical moments are equated to observed sample moments, i.e.,

$$E(X) = 1.25 (1 - \alpha) Sp^* + \frac{(1 - Sp^* + \alpha Sp^*) (1 - 1.25Sp^* + 1.25\alpha p^*)}{p^* (1 - \alpha)} \tag{8}$$

$$E(X^2) = (1.25)^2 (1 - \alpha) Sp^* + (1 - Sp^* + \alpha Sp^*) \left[ \frac{5.06}{p^* (1 - \alpha)} + \frac{(1 - p^* + \alpha p^*)}{[p^* (1 - \alpha)]^2} [2 - 1.56p^* (1 - \alpha)] \right] \tag{9}$$

Solving the equations (8) and (9) one can get the estimates of  $S$  and  $p^*(1 - \alpha)$ . Here it is to be mentioned that it is difficult to estimate the parameters  $\alpha$  and  $p^*$  separately. This problem has arisen in several similar models. So we have estimated  $p^*(1 - \alpha)$  and hence for the known values of  $a$ , the estimate of  $p$  can be obtained.

The proposed model is applied to the same set of data as mentioned in the previous section. The observed and expected values of the number of migrated couples according to length of first closed birth interval are given in Table 2. The value of  $X^2$  is insignificant at 5% level of significance. This shows that the observed pattern of first closed birth interval is described well by the proposed model. The estimate of fecundability obtained in this paper is based on the first closed birth interval which may be considered for the females aged less than or equal to 30 years. In practice it may be seen that the females upto the age of 30 years may give two births particularly in developing countries like India. This implies that such female are highly responsible for first closed birth interval.

TABLE 2 : OBSERVED AND EXPECTED DISTRIBUTION OF FEMALES ACCORDING TO TIME FROM FIRST BIRTH TO SECOND BIRTH WHOSE FIRST BIRTH IS AFTER MIGRATION

Time (in years)	Average (in years)		Number of couples	
			Observed	Expected
0.75	- 1.75	1.25	9	9.34
1.75	- 2.75	2.25	38	42.67
2.75	- 3.75	3.25	25	23.83
3.75	- 4.75	4.25	15	13.31
4.75	- 5.75	5.25	9	7.44
$\geq$	5.75	6.25	10	9.41
Total			106	106.00

^

$S = 0.1996$ ;  $p'(1 - \alpha) = 0.4414$ ;  $X^2 = 1.1596$ ; d.f. = 3

The estimates of  $S$  and  $p^*(1 - \alpha)$  are found to be 0.1996 and 0.4414 respectively. Thus, if we take  $a = 0.10$ , then the estimate of the level of fecundability comes out to be 0.297 for  $y = 2$  months and 0.317 for  $y = 1.8$  months. These estimates are higher than the estimates ( $p = 0.253$  for  $y = 2$  months and  $p = 0.28$  for  $y = 1.8$  months) obtained by Kushwaha (1992) and these are consistent with the estimates ( $p = 0.30$  for females  $< 30$  years and  $p = 0.28$  for females  $\geq 30$  years) obtained by Yadava *et al.* (1992) based on the number of births by these couples. Thus, it may be noted that the present methodology and the methodology developed by Kushwaha are same only the difference exists due to ignorance of foetal wastage by Kushwaha (1992). Further, the level of fecundability obtained in this study and by Yadava *et al.* (1992) shows its reliability and hence the reliability of two different methodologies.

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**APPENDIX****The Data**

"Rural development and population growth (RDPG)—A Sample Survey 1978" was conducted during March-October 1978, from the rural areas of Eastern Uttar Pradesh, India. The main objective of the survey was to study the existing level of fertility, mortality and migration in three types of villages representing different levels of social and economic development. The survey included around 3500 households from the rural areas.

In addition to the detailed survey carried out in 1978, the selected households were also visited in different rounds of six months duration in which data on number of births, birth intervals under different parities, duration of visits by migrated persons to the households were also collected. Such information was obtained for a period of three years (1978-1981) from the reference date of the survey. As a result of this, it has become possible to know the average duration of stay of migrants per year in the household. These information have been utilized to estimate the level of fecundability for such couples.

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