Demography India
Vol. 27, No. 1 (1998), pp. 205-211

R. C. Yadava* and Meenakshi Srivastava**

Extent of Infecundity Derived from Open Birth Interval Data

Introduction

THE quantum and tempo of fertility can be explained if we specify the processes; first how females space their offsprings and second, how many females of a given parity proceed to the next. These processes more often-than-not, relate to the familial practices. It is apparent that the aggregate results of these individual decisions will regulate the future course of fertility.

The Closed Birth Intervals (CBI), are taken as indicators of reproductive performances of those females who are still in the process of reproduction i.e., in the first process. These CBI can be used for estimation of different parmeters such as fecundability, incidence of foetal wastages, post-partum amenorrhea etc., through the use of appropriate models. Nevertheless, such data do not elucidate the proportion of females of any parity who do not proceed to the next birth. Thus, two populations having same CBI patterns may have different fertility rates if their parity progression ratios (PPRs) are different. Therefore in the study of fertility, estimations of PPRs may be of great use in detecting current changes in fertility among the populations.

Recently number of procedures have been advocated by number authors to estimate PPRs under different sets of assumptions and according to type of data available (see Feeny, 1983, 1985; Feeny and Ross, 1984; Pandey and Suchindran, 1985; Yadava and Bhattacharya, 1985; Yadava and Saxena, 1989; Yadava *et. al.*, 1992; etc).

Srinivasan (1967, 1968) proposed a method to estimate instantaneous parity progression ratio (IPPR). There is a conceptual distinction between PPR and IPPR. Infact PPR gives the probability of progression from z'th to (/ + 1)th birth for females who ever had the zth birth

* Professor, Department of Statistics, Faculty of Science, Banaras Hindu University, Varanasi-221005.
** Lecturer, Department of Statistics andComputer Applications, Institute of Social Sciences, Dr. B. R. Ambedkar University, Agra-282004.

in their reproductive life, whereas IPPR gives the probability of progression from parity i to (i + l)th for females who are of parity *i* at the time of survey. Such females (in the context of IPPR) are of different open birth intervals at the time of the survey. Now a natural question arises: Do the females of a specific parity at the time of the survey but of different open birth interval have the same probability of progression to the next parity or are these probabilities different? The intuition says: Perhaps these probabilities may not be same and the probability of progression for females with larger open birth interval may be smaller than the probability of progression for the females with shorter open intervals.

The objective of the present paper is to develop a methodology to estimate the probability of progression from parity *i* to *(i* + 1) for females of parity / having open birth interval in a specified period say (C_1, C_2) at the time of the survey. This may be considered as equivalent to the estimation of proportion of fecund females or alternatively, proportion of infecund or sterile (voluntary or involuntary) females, among such females. These findings may have important policy implications. Should the proportion of fecund females in different open birth interval groups be different it is not advisable to concentrate much of our efforts in terms of family planning on the groups in which proportions of fecund females are substantially less.

A description of the proposed methodology is presented in the next section followed by the estimation of proportion of fecund females in different open birth interval groups. This procedure is essentially based on the extension of procedure of Yadava *et. al.,* (1992). The methodology is applied to on observed set of data taken from a demographic survey.

Methodology

Let α_i and α_i^* denote respectively the values of PPR and IPPR for parity *i*. Srinivasan (1967) has given a procedure to estimate the value of α_i^* utilising data on open birth interval (U_i'), closed birth interval (T_i) and the interval between the last birth and the survey data (V_i) for the females with completed fertility, for whom *i*th birth happens to be the last birth. The basic concept in Srinivasan's procedure is that *i*th births are uniformly distributed over time and the females of parity *i* at the time of survey are of two types: One who are definitely going to proceed to the next birth after the birth of *i*th child and the other who are not capable of proceeding to the next birth. These two types of females may be called as 'fertile; and sterile' females. If U_i denotes the open birth interval for fertile females and S_i denotes open birth interval for sterile females then the basic equation for the estimation of α_i^* is

$$E(U_i') = \alpha_i^* \; E(U_i) + (1 - \alpha_i^*)\, E(S_i) \tag{1}$$

Srinivasan has shown that

$$E(U_i) = \frac{E(T_i^2)}{2E(T_i)}$$

and

$$E(S_i) = \frac{E(V_i^2)}{2E(V_i)}$$

Thus α_i^* can be easily estimated provided that data on U_i', T_i and V_i are available. Recently, Yadava and Saxena (1989) have tried to make distinction between PPR and IPPR more clear and have given an expression which is useful in studying the inter-relationship between PPR and IPPR. The expression is

$$\alpha_i^* = \frac{\alpha_i\, E(T_i)}{\alpha_i\, E(T_i) + (1-\alpha_i)\, E(V_i)} \tag{2}$$

This clearly indicates that α_i^* (IPPR) is always smaller than α_i (PPR). Further, two populations with same value of α_i may have different α_i^* if their $E(T_i)$ and $E(V_i)$ are different. The data needs are same as that required in Srinivasan's procedure.

Yadava and Bhattacharya (1985) proposed a methodology which provides estimate of α_i (PPR). The procedure is developed by including only those females in the study who have open interval less than equal to a period C where C is such that $P[T_i \geq C] = 0$, C can be any value which satisfies the above conditions. The equation used for this purpose is

$$\alpha_i = \frac{C^2 - 2CE(U_i^*)}{2E(U_i^*)[E(T_i) - C] + C^2 - E(T_i^2)} \tag{3}$$

Where $E(U_i)$ denotes mean open birth interval for those females of parity i who have open birth interval less than or equal to C.

While deriving the above equation, Yadava and Bhattacharya (1985) have given an equation, which gives an estimate of the proportion of the fecund females among the females having open interval (O, C). This proportion is denoted as α_i^* (O, C) and it's value is

$$\alpha_i^*(O,C) = \frac{\alpha_i\, E(T_i)}{\alpha_i\, E(T_i) + (1-\alpha_i)\, C} \tag{4}$$

Though specifically not mentioned, this value of α_i^* (O, C) may be interpreted as the value of instantaneous parity progression ratio for the females with open interval (O, C). The choice of C with the condition that $P[T_i \geq C] = 0$ is taken mainly to get the value of the $\int_0^C [1 - F_i(t)]\, dt = E(T_i)$. Here $F_i(t)$ is the distribution function of T_i.

Recently, Yadava *et al.* (1992) have tried to relax the condition on the choice of the value of C. They have argued that $\int_0^C [1 - F_i(t)]\, dt$ can be evaluated easily by using any suitable quadrature formula provided the distribution of T_i (and hence the distribution function of T_i) is known. Without the condition $P[T_i \geq C] = 0$. Using this idea, they have tried to estimate α_i^* (O, C) by the equation

$$\alpha_i^*(O,C) = \frac{\alpha_i \int_0^C [1 - F_i(t)]\, dt}{\alpha_i \int_0^C [1 - F_i(t)]\, dt + (1-\alpha_i)\, C} \tag{5}$$

Here C may be such that $P[T_i \geq C]$ may be more than zero also.

The above equation (5) can be used to estimate the proportion of fecund females among the females with any open birth interval group say (C_1, C_2).

If the number of females included in the study with open birth interval (O, C) is denoted as $N_i(O, C)$, then

$$N_i(O, C) = K_i\,[\alpha_i \int_0^C [1 - F_i(t)]\,dt + 1\,(1 - \alpha_i)\,C] \tag{6}$$

where K_i is some constant depending upon the value of number of females of parity i at the time of the survey.

Among these females, $K_i\,\alpha_i \int_0^C [1 - F_i(t)]\,dt$ will be 'fertile' while $K_i\,(1 - \alpha_i)\,C$ will be 'sterile'.

As mentioned earlier the main objective of the present paper is to develop a methodology to estimate the proportion of fecund or alternatively, infecund females among the females with open interval (C_1, C_2). Precisely, we have utilised the above two equations (5) and (6) to get the estimates of $\alpha_i^*\,(C_i, C_2)$. Thus, substituting the value C as C_i and C_2 in the expressions (5) and (6) we get the values of $\alpha_i^*\,(O, C_1)$, $\alpha_i^*\,(O, C_2)$, $N_i\,(O, C_1)$ and $N_i\,(O, C_2)$. Thus the number of females included in the study and having open interval between C_1 and C_2, $C_1 < C_2$ will be $N_i\,(O, C_2) - N_i\,(O, C_1)$. Let the proportion of fecund females among these females be denoted as $\alpha_i^*\,(C_1, C_2)$ then we have the relation.

$$\alpha_i^*\,(O, C_2) = \frac{N_i\,(O, C_1)\,\alpha_i^*\,(O, C_1) + [N_i\,(O, C_2) - N_i\,(O, C_1)]\,\alpha_i^*\,(C_1, C_2)}{N_i\,(O, C_2)} \tag{7}$$

After simplification we get

$$\alpha_i^*\,(C_1, C_2) = \frac{\alpha_i^*(O, C_2)\,N_i\,(O, C_2) - \alpha_i^*\,(O, C_1)\,N_i\,(O, C_1)}{N_i\,(O, C_2) - N_i\,(O, C_1)} \tag{8}$$

where

$$\alpha_i^*\,(O, C_j) = \frac{\alpha_i\,K_i \int_0^{C_j} [1 - F_i(t)]\,dt}{\alpha_i\,K_i \int_0^{C_j} C_j\,[1 - F_i\,(t)]\,dt + K_i\,C_j\,(1 - \alpha_i)}$$

$$N_i\,(C, C_j) = K_i \left[\alpha_i \int_0^{C_j} .[1 - F_i\,(t)]\,dt + (1 - \alpha_i)\,C_j\right]$$

(j takes values 1 and 2).

Thus, the value of $\alpha_i^*\,(C_1, C_2)$ can be obtained provided the value of α_i and the distribution of T_i is known. It should be noted here that $1 - \alpha_i^*\,(C_1, C_2)$ will give proportion of infecund females, with open birth interval (C_1, C_2). (Since K_i occurs both in the numerator and in denominator, hence it cancels and its value is not needed in the computation of $\alpha_i^* (C_1, C_2)$. Utilizing the data on open and closed birth intervals, the estimate of α_i can easily be obtained using the method suggested by Yadava and Bhattacharya (1985) or the method discussed in Yadava *et al.* (1992). Thus the data requirements are essentially the same (the open and closed birth intervals for different parities) to compute the value of $\alpha_i^* (C_1, C_2)$.

In the next section we have tried to examine the behaviour the values of $\alpha_i^* (C_1, C_2)$ for different values of C_1 and C_2 taking birth interval (T_i) values of α_i and some observed distributions of open and closed birth intervals from demographic survey.

Estimation of Proportion of Fecund Females among Females with Open Interval (C_1, C_2)

A demographic survey was conducted in the year 1978 under the auspices of Centre of Population Studies, Banaras Hindu University in which data about 3500 households from rural areas near Varanasi were obtained. In this survey, apart from other information, data on open and last closed birth intervals were obtained for each currently married female in the age group (15-49) years at the time of survey. The data of survey are utilized for illustration purpose of the methodology.

Yadava *et al.* (1992), have given the distribution of an open and closed birth intervals for different parities and the corresponding estimates of PPRS, utilizing the data of the above survey.

Utilizing the suggested methodology, the estimates of $\alpha_i^* (O, C_1)$ and $\alpha_i^* (C_1, C_2)$ have been obtained for some selected values of C_1 and C_2 for different parities. These are presented in Table 1.

A critical review of the figures suggest that the proportion of fecund females having open birth interval between 5 and 10 years are relatively small for parities 3 and above. For example for parity 3, it is around 30 per cent while for parity 6, it is about 9 per cent. If we examine these figures for open interval between 7 and 10 years the proportion of fecund females becomes substantially small. These figures for parity 3 and 6 are only 4.5 per cent and 1.3 per cent respectively which implicitly imply that proportion of fecund females among females with open interval more than 7 years is very small. This suggests that any attempt to motivate such females for family planning may not yield any significant reduction in fertility in the population. It was perhaps the main reason for not getting substantial reduction in fertility even with larger number of sterilization in sixties and seventies in India.

It is, however, important to mention that many of the women with longer open interval might have achieved this state by some kind of voluntary control (say by limiting their frequency of intercourse or some other method of family planning). Thus these women may need the help of family planning programme so as to continue their status of prolonged open birth interval and such help should ethically be provided to them.

The model incorporated in this paper has, among others, two significant assumptions. Firstly, it is assumed that there is no fertility decline over time. Secondly, that a female after giving a birth may remain either fecund or sterile throughout the observational period, by any voluntary or involuntary methods. However, these two assumptions may not be true in reality. The fertility may decline over time; and, as well as, a female after birth may remain fecund

TABLE 1 : ESTIMATES OF PROPORTION OF FECUND FEMALES FROM OBSERVED DISTRIBUTION OF CLOSED BIRTH INTERVAL, FOR DIFFERENT OPEN BIRTH INTERVAL GROUPS

Parity	α_i	*Interval* (O, C_i) *(in months)*	$\int_0^{C}[1-F_i(t)]dt$	$\alpha_i^*(O, C_i)$	*Interval* (C_i, C_j) *(in months)*	$\alpha_i^*(C_i, C_j)$
		0-60	35.00	0.9932	60-84	0.9364
2	0.996	0-84	36.42	0.9908	60-120	0.8948
		0-120	37.05	0.9872	84-120	0.2750
3		0-60	35.24	0.9178	60-84	0.4563
	0.950	0-84	36.30	0.8914	60-120	0.2852
		0-120	36.50	0.8525	84-120	0.0452
4		0-60	35.94	0.9114	60-84	0.3176
	0.945	0-84	36.59	0.8821	60-120	0.1787
		0-120	36.70	0.8401	84-120	0.0259
5		0-60	35.44	0.7783	60-84	0.2079
	0.856	0-84	36.50	0.7209	60-120	0.1286
		0-120	36.93	0.6466	84-120	0.0371
6		0-60	35.53	0.7336	60-84	0.1809
	0.823	0-84	36.67	0.6699	60-120	0.0915
		0-120	36.83	0.5880	84-120	0.0133
7		0-60	34.84	0.7163	60-84	0.1455
	0.813	0-84	35.78	0.6494	60-120	0.0775
		0-120	36.00	0.5660	84-120	0.0147
8		0-60	31.84	0.4463	60-84	0.0383
	0.603	0-84	32.47	0.3699	60-120	0.0169
		0-120	32.52	0.2916	84-120	0.0012

for some time and then turn sterile afterwards, permanently or temporarily. The initial two assumptions were purposively taken in order to initiate a primary study in this area. Nevertheless, a more comprehensive research is required even with modified assumptions and objectives.

The present authors contribution is, therefore, limited to helping in formulating a suitable methodology for futuristic applications. It is expected that such a methodology with necessary change, as indicated in the preceding paragraph may help in formulating a feasible policy of birth control on the basis of inducation of large birth intervals.

The present paper indicates the limitations of utilising a past data as no fresh data in this regard is in hand. However, this application has enough validity in comprehending a proportion of fecund females among females with specified open birth interval group; the objective of the present research.

References

Feeney, G., 1983, Population dynamics based on birth intervals and parity progression. *Population Studies, 37:* 75-89.

Feeney, G., 1985, Parity progression projection. *Proceedings of the Twentieth General Conference of International Union for the Scientific Study of Population,* Florence, Italy, 4, pp. 125-136.

Feeney, G. and Ross, J. A., 1984, Analysing open birth interval distributions, *Population Studies,* 38: 473-478.

Pandey, A. and Suchindran, C. M., 1989, Pace, Speed and Quantum of Fertility: A Model Based Estimation. *In:* K. Srinivasan, and K. B. Pathak, (eds.), *Dynamics of Population and Family Welfare,* ser. 6. Himalaya Publishing House, Mumbai, pp. 268-279.

Singh, S. N. and Yadava, R. C., 1977, A generalised probability model for an equilibrium birth process. *Demography India,* 6: 163-173.

Srinivasan, K., 1967, Analytical Models for two Types of Birth Intervals with Application to Indian Population. *Thesis* submitted for Doctor of Philosophy in Demography, University of Kerala, Trivandrum, India.

Srinivasan, K., 1968, A set of analytical models for the study of open birth intervals. *Demography, 5:* 34-44.

Yadava, R. *C.* and Bhattacharya, M., 1985, Estimation of Parity Progression Ratios from Closed and Open Intervals Data, Mimeo Centre of Population Studies, Banaras Hindu University, Varanasi, India.

Yadava, R. C. and Saxena, N. C., 1989, The estimation of parity progression and instantaneous parity progression ratios. *In:* S.N. Singh, M.K. Premi, P. S. Bhatia and Ashish Bose, (eds.). *Population Transition in India,* 1. B. R. Publishers, New Delhi, pp. 357-362.

Yadava, R. C., Pandey, A. and Saxena, N. C., 1992, Estimation of parity progression ratios from truncated distribution of closed and open birth intervals. *Mathematical Biosciences,* **110** (2): 181-190.