

R. C. Yadava*, K. N. S. Yadava* and V. S. Singh**

Population Projection Under a Changed Regime of Fertility

Introduction

THE estimation of size of population at any future time has been of considerable interest to planners and researchers and in recent years much attention has been paid to this topic by many of the researchers (Keyfitz, 1975; Mitra, 1976; Frauenthal, 1975; Yadava, 1985; Yadava *et al.*, 1989 and others). They have derived their expressions under varying sets of assumptions mainly utilizing the results of stable population. It is well known that a population that is governed by a regime of unchanging fertility and mortality schedules for a long time, with no migration, ultimately becomes stable. The age structure of this population remains fixed and the size changes with a constant rate of increase, which depends on fertility and mortality schedules of the population concerned. If the fertility and mortality schedules are such that the rate of increase become zero, then the size of the population also becomes fixed and such a population is known as stationary population.

Keyfitz (1975) has shown how the quantitative impacts of sterilization, mortality and emigration (all assumed to take place at a particular age a) on the fertility reduction can be assessed by means of reproductive values. Further, Frauenthal (1975) has worked out some formula for birth trajectory under changing fertility conditions by utilizing Lotka's (1939) renewal equation. Yadava (1985) has derived a formula for population size projection upto the lower limit of the reproductive period (denoted as a) by assuming an abrupt change in the fertility function and utilizing Frauenthal's (1975) equation for birth trajectory. Again, Yadava and Singh (1989) obtained an approximate formula for population size projection obtained an approximate formula for population size projection for a duration twice the

*R. C. Yadava, Professor, K. N. S. Yadava. Reader. Department of Statistics, Banaras Hindu University. Varanasi-221 005.

**V. S. Singh, Lecturer & Head, Department of Statistics. H. N. B. Garhwal University, S. R. T. Campus. Tehri-249001.

lower limit of the reproductive period (i.e., 2α) by exploiting graduation formula for the net maternity function. In all these investigations, the initial population was assumed to be stable and the fertility schedule, $m(a)$ was assumed to be changed as $\rho m(a)$ where ρ is a constant.

At the time of applications some researchers have taken the value of ρ as $\frac{1}{Ro}$ while others have taken as $\frac{Ro^*}{Ro}$ where $1 < Ro^* < Ro$ where Ro represents the net reproduction rate prior

to change in fertility schedule and Ro^* represents the new net reproduction rate after the change in fertility schedule. Later on Yadava *et al.* (1989) derived expressions for projecting the population upto a period α by assuming the changed fertility schedule as $e^{-ra} m(a)$ where r represents the growth rate of the stable population prior to the change in the fertility schedule. Obviously the population generated under this new fertility schedule will ultimately become stationary as the new net reproduction rate will become one.

The objective of the present paper is to derive expressions for projecting the population upto time $t \leq 3\alpha$ allowing the similar change in fertility schedule (i.e., $e^{-ra} m(a)$) as considered in Mitra (1976) and Yadava *et al.* (1989). The expressions for birth trajectory and population sizes are worked out and their illustration with one example is given.

Expressions for Birth Trajectory and Population Size at Time t ($t \leq 3\alpha$)

In most of the population projection problem, without loss of generality, the size of the initial population is taken as unity and we consider the same concept here also. Lotka (1939) has given a renewal equation for female births $B(t)$ at time t as:

$$B(t) = \int_t^\infty c(a-t) \frac{p(a)}{p(a-t)} m(a, t) da + \int_0^t B(t-a) p(a) m(a, t) da \quad t > 0 \quad (1)$$

For simplicity let us denote

$$G(t) = \int_t^\infty c(a-t) \frac{p(a)}{p(a-t)} m(a, t) da$$

where $c(a)$ is the age distribution at time $t = 0$, $p(a)$ the fraction of female population that survives upto age a , $m(a) da$ is the probability that a female who is of age a will bear a female child in the next da period of her life.

However, if $p(a)$ remains unchanged and $m(a, t)$ is changed to $m(a, t) = e^{-ra} m(a)$ then the derivations for expression $B(t)$ require modifications. It is obvious that for $0 \leq t \leq \alpha$, $\int_0^t B(t-a) m(a) p(a) da$ is zero because in this situation the integrand is zero for the whole range as $m(a)$ is zero for $a \leq \alpha$.

Thus if the initial population is assumed to be stable with growth rate r then $c(a) = boe^{-ra} p(a)$ where bo is the birth rate at time $t = 0$.

Then for $0 \leq t \leq \alpha$, the expression for $G(t)$ will become

$$\begin{aligned}
 G(t) &= \int_{\alpha}^{\beta} boe^{-r(a-t)} p(a-t) \frac{p(a)}{p(a-t)} e^{-ra} m(a) da \\
 &= boe^{rt} \int_{\alpha}^{\beta} e^{-2ra} p(a) m(a) da
 \end{aligned} \tag{2}$$

where β is the higher age of non-zero fertility i.e., β represents the end of reproductive period.

Mitra (1976) has shown that under certain general conditions the expression

$$\int_{\alpha}^{\beta} e^{-2ra} p(a) m(a) da \simeq \frac{e^{r^2\sigma^2}}{Ro} \tag{3}$$

where σ^2 is the age variance of net fertility schedule and Ro is the net reproduction rate in the initial population.

Thus

$$\begin{aligned}
 B(t) &= boe^{rt} \frac{e^{r^2\sigma^2}}{Ro} \quad \text{for } 0 \leq t \leq \alpha \\
 &= boe^{rt} \cdot K \quad \text{where } K = \frac{e^{r^2\sigma^2}}{Ro}
 \end{aligned} \tag{4}$$

Further if $\alpha \leq t \leq 2\alpha$ then

$$\begin{aligned}
 G(t) &= \int_{t}^{\beta} boe^{-r(a-t)} \cdot P(a-t) \frac{p(a)}{p(a-t)} e^{-ra} m(a) da \\
 &= \int_{\alpha}^{\beta} boe^{rt} \cdot e^{-2ra} p(a) m(a) da - \int_{\alpha}^t boe^{rt} e^{-2ra} p(a) m(a) da \\
 &\simeq boe^{rt} K - boe^{rt} \int_{\alpha}^t e^{-2ra} p(a) m(a) da
 \end{aligned} \tag{5}$$

It is quite difficult to obtain exact expression for $\int_{\alpha}^t e^{-2ra} p(a) m(a) da$. However, the integral may be solved approximately utilizing certain suitable quadrature formula with the knowledge of fertility and mortality schedules as well as the value of growth rate. Alternatively an approximation for integral

$\int_{\alpha}^t e^{-2ra} p(a) m(a) da$ may be taken as

$$\int_{\alpha}^t e^{-2ra} p(a) m(a) da \simeq \frac{t - \alpha}{\beta - \alpha} \cdot \frac{e^{r^2\sigma^2}}{Ro} = \frac{t - \alpha}{\beta - \alpha} \cdot K \tag{6}$$

Taking this approximation

$$\begin{aligned}
 G(t) &= boe^{rt} \cdot K - boe^{rt} \cdot K \frac{t - \alpha}{\beta - \alpha} \\
 &= boe^{rt} \cdot K \left[\frac{\beta - t}{\beta - \alpha} \right]
 \end{aligned} \tag{7}$$

Now our problem is to evaluate $\int_{\alpha}^t B(t-a)e^{-ra} p(a) m(a) da$. Though in the context of the initial stable population $B(t-a) = e^{-r(t-a)}$ but here due to change in the fertility schedule, the expression for $B(t-a)$ will be changed.

Obviously for $\alpha \leq t \leq 2\alpha$, $0 \leq t-a \leq \alpha$ and for $\alpha \leq a \leq t$ and for this situation $B(t-a) = boe^{r(t-a)} \times K$.

Thus

$$\begin{aligned} \int_{\alpha}^t B(t-a)e^{-ra} p(a)m(a)da &= \int_{\alpha}^t boe^{rt} \cdot K e^{-2ra} p(a)m(a)da \\ &\approx boe^{rt} \times K^2 \cdot \frac{t-\alpha}{\beta-\alpha} \end{aligned} \quad (8)$$

Thus finally for $\alpha \leq t \leq 2\alpha$.

$$\begin{aligned} B(t) &= boe^{rt} \cdot K \left(\frac{\beta-t}{\beta-\alpha} \right) + boe^{rt} \cdot K^2 \frac{t-\alpha}{\beta-\alpha} \\ &= B^*(t) \quad (\text{say}) \end{aligned} \quad (9)$$

For $2\alpha \leq t \leq 3\alpha$, as derived earlier, we get

$$G(t) = boe^{rt} \cdot K \left(\frac{B-t}{\beta-\alpha} \right) \quad (10)$$

Now our problem is to evaluate the integral

$$\int_{\alpha}^t B(t-a)e^{-ra} p(a)m(a)da$$

We know that

$$\begin{aligned} \int_{\alpha}^t B(t-a)e^{-ra} p(a)m(a)da &= \int_{\alpha}^{t-\alpha} B(t-a)e^{-ra} p(a)m(a)da \\ &\quad + \int_{t-\alpha}^t B(t-a)e^{-ra} p(a)m(a)da \end{aligned} \quad (11)$$

It is obvious that for $2\alpha \leq t \leq 3\alpha$, $t-a$ will lie between α and 2α for $\alpha \leq a \leq t-\alpha$ and for this situation expression for $B(t)$ derived in equation (9) for $\alpha \leq t \leq 2\alpha$ may be substituted. Further $t-a$ will lie between 0 and α for $t-\alpha \leq a \leq t$ and for this situation expression for $B(t)$ $0 \leq t \leq \alpha$ may be substituted. Thus

$$\begin{aligned} \int_{\alpha}^t B(t-a)e^{-ra} p(a)m(a)da &= \int_{\alpha}^{t-\alpha} boe^{r(t-a)} \cdot K \left(\frac{\beta-t+a}{\beta-\alpha} \right) e^{-ra} p(a)m(a)da \\ &\quad + \int_{\alpha}^{t-\alpha} boe^{r(t-a)} \cdot K^2 \left(\frac{t-a-\alpha}{\beta-\alpha} \right) e^{-ra} p(a)m(a)da \\ &\quad + \int_{t-\alpha}^t boe^{r(t-a)} \cdot K e^{-ra} p(a)m(a)da \end{aligned} \quad (12)$$

Taking the approximations as earlier, the above expression can be approximated as

$$\begin{aligned} &= boe^{rt} \cdot K^2 \frac{\beta-t}{\beta-\alpha} \frac{t-2\alpha}{\beta-\alpha} + boe^{rt} \cdot K^3 \frac{t-\alpha}{\beta-\alpha} \frac{t-2\alpha}{\beta-\alpha} + boe^{rt} \cdot K^2 \frac{\alpha}{\beta-\alpha} \\ &+ \int_{\alpha}^{t-\alpha} boe^{rt} \cdot K \frac{ae^{-2ra}}{\beta-\alpha} p(a)m(a)da - \int_{\alpha}^{t-\alpha} \frac{boe^{rt} K^2}{\beta-\alpha} ae^{-2ra} p(a)m(a)da \end{aligned} \quad (13)$$

It seems quite difficult to obtain exact expression $\int_{\alpha}^{t-\alpha} ae^{-2ra} p(a)m(a)da$. However, this can be approximated by considering it as

$$\begin{aligned} \int_{\alpha}^{t-\alpha} ae^{-2ra} p(a)m(a)da &= \frac{\int_{\alpha}^{t-\alpha} ae^{-2ra} p(a)m(a)da}{\int_{\alpha}^t e^{-2ra} p(a)m(a)da} \int_{\alpha}^{t-\alpha} e^{-2ra} p(a)m(a)da \\ &\approx \frac{t}{2} \cdot K \cdot \frac{t-2\alpha}{\beta-\alpha} \end{aligned} \quad (14)$$

Under these approximations, finally, for $2\alpha \leq t \leq 3\alpha$, we get

$$\begin{aligned} B(t) &= boe^{rt} \cdot K \frac{\beta-t}{\beta-\alpha} + boe^{rt} \cdot K^2 \frac{\beta-t}{\beta-\alpha} \frac{t-2\alpha}{\beta-\alpha} \\ &+ boe^{rt} \cdot K^3 \frac{t-\alpha}{\beta-\alpha} \frac{t-2\alpha}{\beta-\alpha} + boe^{rt} \cdot K^2 \frac{\alpha}{\beta-\alpha} \\ &+ boe^{rt} \cdot K^2 \frac{t}{2(\beta-\alpha)} \frac{t-2\alpha}{\beta-\alpha} - boe^{rt} \cdot K^3 \frac{t}{2(\beta-\alpha)} \frac{t-2\alpha}{\beta-\alpha} \\ &= boe^{rt} \cdot K \left[\frac{\beta-t}{\beta-\alpha} + K \cdot \frac{(\beta-t)(t-2\alpha)}{(\beta-\alpha)^2} + K^2 \frac{(t-\alpha)(t-2\alpha)}{(\beta-\alpha)^2} \right. \\ &\left. + K \frac{\alpha}{\beta-\alpha} + K \cdot \frac{t(t-2\alpha)}{2(\beta-\alpha)^2} - K^2 \frac{t(t-2\alpha)}{2(\beta-\alpha)^2} \right] \quad \text{say} \quad (15) \\ &= B^{**}(t) \end{aligned}$$

Thus expressions for $B(t)$, for three ranges of t , viz. $0 \leq t \leq \alpha$, $\alpha \leq t \leq 2\alpha$ and $2\alpha \leq t \leq 3\alpha$ are obtained vide equations (4), (9) and (15), respectively.

Once the value of $B(t)$ is known, the population at any time t can easily be calculated. In fact, the population at any time t will be sum of two components viz., (i) Survivors of the initial population (observed at time $t = 0$) upto time t and (ii) Survivors of the births occurred during $(0, t)$.

Let us consider part (i):

For population at age y of the initial stable population is $boe^{-ry} p(y)$ and the probability that they will survive upto time t is $\frac{p(t+y)}{p(y)}$. Therefore, the survivors of the initial population upto time t is

$$\int_0^w boe^{-\tau y} p(y) \frac{p(t+y)}{p(y)} dy$$

i.e., $\int_0^w boe^{-\tau y} p(t+y) dy$ (16)

where w is the highest age of life.

Now putting $t+y=z$, the above expression becomes

$$boe^{\tau t} \int_t^w e^{-\tau z} p(z) dz$$

Now

$$\begin{aligned} boe^{\tau t} \int_t^w e^{-\tau z} p(z) dz &= e^{\tau t} \int_0^w boe^{-\tau z} p(z) dz - e^{\tau t} \int_0^t boe^{-\tau z} p(z) dz \\ &= e^{\tau t} - e^{\tau t} \int_0^t boe^{-\tau z} p(z) dz \\ \int_0^w boe^{-\tau z} p(z) dz &= 1 \\ &= e^{\tau t} - e^{\tau t} \bar{A}(t) \\ &= e^{\tau t} [1 - \bar{A}(t)] \end{aligned}$$
 (17)

where $\bar{A}(t)$ is the proportion of persons upto age t in the initial stable population. Now our problem is to find the number of survivors out of the births in $(0, t)$ upto time t . If $t \leq \alpha$ then the number of survivors out of the births in $(0, t)$ say $S(t)$ can be obtained as:

$$S(t) = \int_0^t boe^{\tau y} \cdot K p(t-y) dy$$

Now putting $t-y=z$, we get

$$\begin{aligned} S(t) &= \int_0^t boe^{\tau(t-z)} K p(z) dz \\ &= e^{\tau t} K \int_0^t boe^{-\tau z} \cdot p(z) dz \\ &= e^{\tau t} K \bar{A}(t) \end{aligned}$$
 (18)

Thus for $t \leq \alpha$, $p(t)$, the population size at time t is

$$p(t) = e^{\tau t} [1 - \bar{A}(t)] + e^{\tau t} K \bar{A}(t)$$
 (19)

For $\alpha \leq t \leq 2\alpha$,

The number of survivors from births in $(0, t)$ to time t will be

$$\begin{aligned} S(t) &= \int_0^\alpha boe^{\tau y} \cdot K p(t-y) dy \\ &+ \int_\alpha^t boe^{\tau y} \cdot K \frac{(\beta-y)}{(\beta-\alpha)} p(t-y) dy \\ &+ \int_\alpha^t boe^{\tau y} \cdot K^2 \frac{(y-\alpha)}{(\beta-\alpha)} p(t-y) dy \end{aligned}$$
 (20)

Putting $t - y = z$ and solving the above expression we get

$$\begin{aligned}
 S(t) &= \int_{t-\alpha}^t b o e^{r t} \cdot K e^{-r z} p(z) dz + \int_0^{t-\alpha} b o e^{r t} \cdot K e^{-r z} \frac{(\beta - t + z)}{(\beta - \alpha)} p(z) dz \\
 &+ \int_0^{t-\alpha} b o e^{r t} \cdot K^2 e^{-r z} \frac{(t - \alpha - z)}{(\beta - \alpha)} p(z) dz \\
 &\approx e^{r t} \cdot K [\bar{A}(t) - \bar{A}(t - \alpha)] + e^{r t} \cdot K \frac{(\beta - t)}{(\beta - \alpha)} \bar{A}(t - \alpha) \\
 &+ \frac{e^{r t} K \bar{A}(t - \alpha) \bar{\mu}(t - \alpha)}{(\beta - \alpha)} \\
 &+ e^{r t} \frac{K^2 \bar{A}(t - \alpha)}{(\beta - \alpha)} [(t - \alpha) - \bar{\mu}(t - \alpha)] \tag{21}
 \end{aligned}$$

where $\bar{\mu}(t - \alpha)$ is the mean age of person upto age $(t - \alpha)$ in the initial stable population. Thus

$$\begin{aligned}
 p(t) &= e^{r t} [1 - \bar{A}(t)] + e^{r t} \cdot K [\bar{A}(t) - \bar{A}(t - \alpha)] \\
 &+ \frac{e^{r t} \cdot K(\beta - t)}{(\beta - \alpha)} \bar{A}(t - \alpha) + \frac{e^{r t} \cdot K \bar{A}(t - \alpha) \cdot \bar{\mu}(t - \alpha)}{(\beta - \alpha)} \\
 &+ \frac{e^{r t} \cdot K^2 \bar{A}(t - \alpha)}{(\beta - \alpha)} [(t - \alpha) - \bar{\mu}(t - \alpha)] \tag{22}
 \end{aligned}$$

Now for $2\alpha \leq t \leq 3\alpha$, the number of survivors from births in (o, t) to time t will be

$$\begin{aligned}
 S(t) &= \int_0^\alpha b o e^{r y} \cdot K p(t - y) dy + \int_\alpha^{2\alpha} b o e^{r y} \cdot K \frac{(\beta - y)}{(\beta - \alpha)} p(t - y) dy \\
 &+ \int_\alpha^{2\alpha} b o e^{r y} \cdot K^2 \frac{(y - \alpha)}{(\beta - \alpha)} p(t - y) dy \\
 &+ \int_{2\alpha}^t B^{**}(y) p(t - y) dy \tag{23}
 \end{aligned}$$

The above integrals can be evaluated easily by putting $t - y = z$ and solving in the manner as described earlier. However the integral $\int_{2\alpha}^t B^{**}(y) p(t - y) dy$ will include terms which will require knowledge of the values of second moment of the ages of persons in different age groups of the initial population. Alternatively since $2\alpha \leq t \leq 3\alpha$ $B^{**}(t)$ may be assumed to be approximately uniformly distributed over the period and the survivors among these births upto time t , may be approximated as

$$S(t) = B^{**}(t)(t - 2\alpha) \frac{L_{(o, t-2\alpha)}}{(t - 2\alpha)l_0}$$

Where $L_{(o, t-2\alpha)}$ represents the usual life table function ($L_{x, x+n}$) for the interval $(o, t - 2\alpha)$.

Thus finally we get

$$\begin{aligned} P(t) = & e^n [1 - \bar{A}(t) + e^n \cdot K [\bar{A}(t) - \bar{A}(t - \alpha)]] \\ & + \frac{e^n K}{(\beta - \alpha)} [\bar{A}(t - \alpha) - \bar{A}(t - 2\alpha)] [(\beta - t) + \bar{\mu}(t - \alpha, t - 2\alpha)] \\ & + \frac{e^n K^2}{(\beta - \alpha)} [\bar{A}(t - \alpha) - \bar{A}(t - 2\alpha)] [(t - \alpha) - \bar{\mu}(t - \alpha, t - 2\alpha)] \\ & + B^{**}(t) \times (t - 2\alpha) \frac{L_{(o, t-2\alpha)}}{(t - 2\alpha)l_0} \end{aligned} \quad (24)$$

An Illustration

The expressions derived in equations (19), (22) and (24) for projecting the population $P(t)$ at time t for three ranges of t viz., $0 \leq t \leq \alpha$, $\alpha \leq t \leq 2\alpha$ and $2\alpha \leq t \leq 3\alpha$, respectively, are illustrated with one example taking some numerical values of the parameters involved in the expressions. For application of the procedure, we need the values of the rate of growth (r), net reproduction rate (R_0), age variance of net maternity function (σ^2), birth rate (b_0) of the initial stable population alongwith its age distribution so that the proportion of persons below a particular age as well as mean ages of persons in various age ranges could be computed. In this context, the population included in the Rural Development and Population Growth (RDPG) Survey (1978) is considered for illustration. On the basis of various characteristics of this population (as well as populations of the other surveys conducted in the same region during the last two decades) it may be safely assumed that the population is approximately stable as there has been no drastic change in the fertility and mortality schedules. Various other authors viz., Yadava (1985), Singh (1989), Yadava *et al.* (1989) have also assumed this population to be stable in their illustrations. The values of $m(a)$ computed from the age specific fertility rate obtained in the RDPG Survey are given in the Appendix.

In the context of this population, the values of μ and σ^2 are computed on the basis of the following expressions

$$\mu = \frac{R_1}{R_0}$$

$$\text{and } \sigma^2 = \left(\frac{R_2}{R_0} \right) - \left(\frac{R_1}{R_0} \right)^2$$

where,

$$R_t = \int_{\alpha}^{\beta} a' p(a) m(a) da$$

The values of μ , σ^2 and R_0 (i.e. $= \int_{\alpha}^{\beta} p(a) m(a) da$) are found to be 28.646, 52.2938 and

2.12 respectively. Once knowing the values of μ and R_0 , the value of r $\left(r = \frac{\ln R_0}{\mu} \right)$ was

found to be 0.026. For the computation of other values viz. $\bar{A}(t)$ and $\bar{\mu}(x, y)$, the characteristics of the stable population corresponding to the value of $r = 0.026$ and mortality level 13 of South Model Life Table (given in Coale and Demeny, 1966) was taken (this mortality level was chosen because the mortality level of the population included in RDPG Survey (1978) was quite close to this level of mortality of the Model Life Table). The values of age distribution ($c(a)$) and birth rate (bo) have been taken corresponding to this stable population. The values of bo is found to be 0.0426. The values of $\bar{A}(t)$ for different values of t are given in Table 1. For example, the value of $\bar{A}(t) \int_0^t boe^{-ra} p(a) da$ = proportion of persons upto age t in the initial stable population) for $t = 5$ came out to be 0.1660.

With the help of these parametric values, the population is projected for the values of $t = 5, 10, 15, 20, 25, 30, 35, 40$ and 45 years. Here the value of α is taken as 15 years so that 3α becomes 45 years and the condition of $t \leq 3\alpha$ is satisfied. As mentioned earlier, the size of the initial population is taken as unity. While finding expression for $P(t)$ for $2\alpha \leq t \leq 3\alpha$, it was assumed that $B^{**}(t)$ was uniformly distributed over the whole interval. However, when we computed the values of $B^{**}(35)$, $B^{**}(40)$ and $B^{**}(45)$ they were found not exactly equal but were approximately equal. Hence the births in the three intervals viz. (i) between $t = 30$ and $t = 35$; (ii) between $t = 30$ and $t = 40$ and (iii) between $t = 30$ and $t = 45$ have respectively been computed as $5B^{**}(35)$, $5B^{**}(35) + 5B^{**}(40)$ and $5B^{**}(35) + 5B^{**}(40) + 5B^{**}(45)$. The life table functions viz. L_{0-5} , L_{0-10} and L_{0-15} have been taken corresponding to the mortality level 13 of the South Model Life Table of Coale and Demeny (1966). The projected

TABLE 1: THE VALUES OF $\bar{A}(t)$ FOR DIFFERENT VALUES OF t (upto 45 years)

Time (t)	Values of $\bar{A}(t)$
5	0.1660
10	0.3014
15	0.4183
20	0.5193
25	0.6061
30	0.6803
35	0.7438
40	0.7979
45	0.8438

TABLE 2: PROJECTED POPULATION AT DIFFERENT POINT OF TIME t

Time (t)	Projected population $P(t)$
5	1.04216
10	1.09706
15	1.16107
20	1.22958
25	1.29664
30	1.36145
35	1.42476
40	1.49258
45	1.53048

*Assuming initial population to be 1.

population $P(t)$ for different values of t are presented in Table 2. For example, the value of $P(t)$ for $t = 10$ is computed from the formula (19) as

$$P(t) = e^{rt} \left[1 - \bar{A}(t) \left(1 - \frac{e^{r^2 \sigma^2}}{Ro} \right) \right] \quad \left(\text{since, } k = \frac{e^{r^2 \sigma^2}}{Ro} \right)$$

$$\begin{aligned} \text{i.e., } P(t)_{t=10} &= e^{10r} \left[1 - \bar{A}(10) \left(1 - \frac{e^{r^2 \sigma^2}}{Ro} \right) \right] \\ &= e^{10 \times 0.026} \left[1 - 0.3014 \left(1 - \frac{e^{(0.026)^2 (52.2938)}}{2.12} \right) \right] = 1.09706 \end{aligned}$$

Similarly, the value of $P(t)$ for $t = 35$ has been computed from the equation (24) as,

$$\begin{aligned} P(t)_{t=35} &= e^{35r} [1 - \bar{A}(35)] + e^{35r} K [\bar{A}(35) - \bar{A}(35 - \alpha)] \\ &+ \frac{e^{35r} \cdot K}{(\beta - \alpha)} [\bar{A}(35 - \alpha) - \bar{A}(35 - 2\alpha)] [(\beta - 35) + \bar{\mu}(35 - \alpha, 35 - 2\alpha)] \\ &+ \frac{e^{35r} \cdot K^2}{(\beta - \alpha)} [\bar{A}(35 - \alpha) - \bar{A}(35 - 2\alpha)] [(35 - \alpha) - \bar{\mu}(35 - \alpha, 35 - 2\alpha)] \\ &+ B^{**}(35) \cdot (35 - 2\alpha) \frac{L_{(\alpha, 35 - 2\alpha)}}{(35 - 2\alpha) l_0} \end{aligned}$$

Putting the values of α , β and K , $B^{**}(35)$ has been found from the equation (15) and after putting it in the above expression, $P(t)_{t=35}$ came out to be 1.42476 times the initial population.

A review of Table 2 indicates that in 15 years the population will become 1.16 times the initial population while in 40 years it will become almost one and half times the initial population. It is to be noted that if there were no change in the fertility schedule then with this value of r (i.e., $r = 0.026$) the population will become double in around 27 years. Further under the new fertility schedule (i.e., $e^{-ra} m(a)$), the net reproduction rate is unity, but the population is not stationary i.e., the size of population is not constant over time. This is due to a characteristic of the population (like moment of inertia in Physics) discussed in Keyfitz (1971), where he has shown that even with a drop in fertility to the replacement level, the population will continue to grow upto certain time and after that it will become stationary.

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Appendix

VALUES OF $p(a)m(a)$ OBTAINED FROM THE RDPG SURVEY 1978

<i>Age</i>	$p(a)$	$m(a)^*$	$p(a)m(a)$
15-19	0.75517	0.32406	0.24472
20-24	0.74104	0.72401	0.53652
25-29	0.72334	0.69204	0.50058
30-34	0.70420	0.60114	0.42332
35-39	0.68450	0.37310	0.25539
40-44	0.66350	0.19646	0.13035
45-49	0.64092	0.07415	0.04753

$$m(a)^* = ASFR \times S(a) \times Q(0) \times 5$$

where,

ASFR = Age Specific Fertility Rate

S(a) = Proportion of eligible couples in the population under study

Q(0) = Sex-ratio at birth