Demography India
Vol. 17, No. 2 (1988), pp. 273-277

P. M. Kulkarni

Measures of Aging : A Proposed Amendment

IN a recent paper, Basu and Basu (1987) have introduced some new measures of ageing which possess certain desirable properties that the head-count ratio, the most commonly used index of ageing, does not. The head-count ratio is simply the proportion of population with an age equal to or exceeding a pre-designated cut-off point. This ratio does not, however, indicate how old the old are. Moreover, it does not have the properties of continuity and monotonicity (Axioms *C* and *M* in the paper of Basu and Basu). In proposing measures which have these properties, Basu and Basu draw upon the literature on the measurement of poverty. These new measures of ageing do not, however, have the range [0, I], as the head-count ratio as well as the analogous poverty measures do. The objective of this note is to point out why this has happened, and to suggest an appropriats amendment.

Some notation is necessary. Following Basu and Basu (1987), *let* $\underline{y}$ = (Y_1.. y_n) represent the population where $y_1 > y_2 > Y_3$ ... are ages of persons, arranged in descending order of age. Let the number of persons be denoted by *n* ($\underline{y}$) and let

$$q(\underline{y}) = \text{Number of old psrsons in population } \underline{y}$$

$$= \text{Number of } y_4 \text{ values} > G,$$

where *G* is the cut-off point for old age, i.e., all persons aged *G* or higher arc classified as 'old'.

The head-count ratio equals

$$H(\underline{y}) = q(\underline{y})/n(\underline{y}).$$

The argument refers to the population y being studied, and, for the sake of simplicity, can be dropped as long as one is referring to a single given population. In that case n and q would give the number of persons and the number of old persons respectively in the population, and $H — q / n$. It is easy to see that H takes values in the range [0, 1]; equals 0 when none is old, and 1 when all are.

The measures introduced by Basu and Basu (1987) are I, P and Q which are stated below by attaching the subscript B as I_B, P_B, and Q_B respectively. The cut-off age of 65 used for illustration in their paper is replaced here by G and the argument dropped. Thus,

$$I_B = \sum_{i=1}^{q} (y_i - G)/(q \cdot G),$$

$$\bar{P}_B = \sum_{i=1}^{q} (y_1 - G)^2/(n \cdot G^2) \text{ and}$$

$$Q_B = H \cdot I_B$$

$$= \sum_{i=1}^{q} (y_i - G)/(n \cdot G) \qquad (1)$$

An implicit assumption in the definition of I_B is that $q > 0$, i.e., at least some persons are classified as old.

The ranges of these indices can be found quite easily. Let ω be the maximum possible age, and, to avoid trivial situations let both G and ω — G be assumed to be strictly greater than 0, i.e, 0<G<ω. Note that I_B, which is the age-gap index, would equal 0 if none is older than G, i.e., all the old persons are of exact age G, and its maximum value would be

$$\sum_{i=1}^{q} (\omega - G)/(q \cdot G) = (\omega - G)/G$$

when all tbe old persons are of age ω, and therefore its range is [0, (ω—G)/G].

Similarly, the range[1] of $\bar{P}_B$ is $[0, (\omega - G)^2/G^2]$ and of Q_B $[0, (\omega - G)/G]$. Clearly, none of the three measures has the range [0, 1] except in the special case $\omega = 2G$.

An index, of course, need not necessarily have the range [0, 1]. But it would certainly be convenient if it does. The poverty measures, of which these ageing measures are analogues, do indeed have the range [0, 1] which makes interpretation of values quite easier.[2] And with appropriate normalisation, the measures of ageing can also be made to fall in this.

To this end, it is proposed to replace I_B, $\bar{P}_B$, and Q_B by

$$I \sum_{i=1}^{q} (y_i - G)/(q \cdot (\omega - G)),$$

$$\bar{P} = \sum_{i=1}^{q} (y_i - G)^2/(n \cdot (\omega - G)^2), \quad \text{and}$$

$$Q = H \cdot I = \sum_{i=1}^{q} (y_i - G) / (n \cdot (\omega - G)), \text{ respectively.} \tag{2}$$

For the sake of completeness, it would be useful to introduce an intermediate measure J as

$$J = \sum_{i=1}^{q} (y_i - G)^2/(q \cdot (\omega - G)^2 \tag{3}$$

so that $\bar{P}$ is the product of H and J. Both I and J can be defined only if $q > 0$.

It can be seen that each of these measures falls in the range [0, 1] Both I and J equal 0 when none of the old is older than G, and 1 when all the old are of age ω. $\bar{P}$ and Q equal 0 when none is old (so that $H = 0$) *or* when no old person is older than G ($I = J = 0$). Both $\bar{P}$ and Q equal 1 when all are old ($H = 1$) *and* all of them are of age ω ($I = J = 1$). I, $\bar{P}$ and Q can be

1. $\bar{P}_B$ will take the highest possible value if all the persons are of age ω. This implies, first, that, all are old, and therefore $q = n$, and $y_i = \omega$ for all i, so that

$$\bar{P}_B = \sum_{i=1}^{q} (y_i - G)^2 / (n \cdot G^2) = \sum_{i=1}^{n} (\omega - G)^2 / (n \cdot G^2) = (\omega - G)^2 / G^2.$$

Similarly, if all are of age ω, Q_B equals $(\omega - G)/G$

2 In fact, a poverty index is often defined as a mapping from x to [0, 1] where x is the set of all alternative social states. See, for example, Basu (1985).

interpreted in exactly the same manner as I_B, P_B and Q_B . I measures how old the old are. It is simply the mean excess over G of the old population, normalised by the maximum value, i.e., (to — G). J also measures the oldness of the old, but rather than the mean excess, it is the mean squared excess (like the mean squared error) over G, and is also normalised. Note that the mean squared excess can be decomposed as :

$$\sum_{i=1}^{q} (y_i - G)^2 / q = \left(\left(\sum_{i=1}^{q} (y_i - \overline{y_G})^2 + q\,(\overline{y_G} - G)^2 \right) \middle/ q \right. = \sigma_G^2 + (y_G - G)^2$$

where $\bar{y}_G$ = mean age of the old $= \sum_{i=1}^{q} y_i/q$ and σ_G^2 = variance of the age of the old $= \sum_{i=1}^{q} (y_i - \bar{y}_G)^2/q$. Therefor, $J = (\sigma_G^2 + (\overline{y_G} - G)^2/(\omega - G)^2$, which means that J is the sum of the variance of the age of the old, and the squared excess of their mean age over G, normalised by dividing the sum by $(\omega - G)^2$. Thus, though both I and J measure the oldness of the old, I reflects the average age by which the old are older than the cut-off point, and J reflects both this average and the variance. $\bar{P}$ and Q reflect both the proportion old, and the oldness of the old measured through mean squared excess in the case of $\bar{P}$, and mean excess in the case of Q.

Though I, $\bar{P}$, and Q (as well as J) have the advantage that they are normalised to fall in the range [0, 1], an apparent disadvantage is that information on ω, the maximum possible age, is needed. There is no universally accepted value for this. Using a value obtained from a given population, like the age of the oldest living person, would not be appropriate because that would make cross-population comparisons difficult. But even the cut-off point G has to be chosen by judgement, and, for comparisons across populations, held constant. Applying the same logic, ω can also be chosen by judgement and held constant. A convenient value for ω would be 100. There are not many persons beyond this age in most contemporary populations. The measures I, J, Q and $\bar{P}$ can then be interpreted as ageing measures relative to a population in which all the old or all the persons are 100 year old. Since the proportion of population over the age 100 is very small, and claims of ages well over this are often highly suspect (see Meyers 1978, for an investigation), little would be lost by treating all such persons as 100 year old. A rather non-technical but none the less important advantage with 100 is that it is widely accepted as a round figure. It should not, hence, be difficult to have a consensus on using it in place of ω.

As an illustration, values of the measures H, Q_B, and $\bar{P}_B$, as well as the modified measures I, J, Q, and $\bar{P}$ are given in Table 1 for the data on India and Japan given by Basu and Basu.

As a population ages, the indexes would also rise. With substantial improvements in health conditions, however, many old persons would no longer appear so old. It would make sense, in such a situation, to raise the cut-off age *G,* as well as the maximum age, to. It can be easily verified that if there is an upward shift in the age, and an equal shift in *G* and to, the indexes remain unchanged. In fact, the indexes remain unchanged under any linear transformation that is applied to ages and simultaneously to *G* and w. Of course, since there is a universally accepted unit for age (year), such a transformation is not needed. But it is a good property to have anyway.

TABLE 1-VALUES OF VARIOUS MEASURES OF AGEING

Index	Japan		India	
	1980	2025	1980	2025
H	.08876	.19508	.03005	.07479
QB	.01160	.03145	.00337	.00891
$\bar{P}_B$	.00234	.00716	.00062	.00170
	.24273	.29945	.20854	.22121
	.09099	.12655	.07064	.07826
Q	.02155	.05842	.00627	.01654
P	.00808	.02469	.00212	.00585

SOURCE : Computed from the data given in Table 1 of Basu and Basu (1987).

Notes : 1. For definitions of the indices, see text.
Z. The assumptions about the average age in each class Interval are the same as those made by Basu and Basu.
3. *G* is assumed to be 65, and *a to* be 100.
4. The first three rows correspond to the figures in Table 2 of Basu and Basu.

References

Basu, Alaka and Kaushik Basu, 1987, The Greying of Populations: Concepts and Measurement, *Demography India,* 16.

Basu, Kausbik, 1935, Poverty Measurement: A Decomposition of the Normalisation Axiom, *Econometrica,* 53.

Meyers, Robert J., 1978, An Investigation of an Alleged Centenarian, *Demography,* **15.**