Demography India
Vol. 17, No. 2 (1988), pp. 278-288

S. Mitra

Effect of Emigration on Other Demographic Measures

Introduction

OF the two components of international migration, immigration, for obvious reasons has a long history of record keeping and careful studies by appropriate agencies of the governments and other interested parties. In contrast, emigration has received no more than a scant attention which, to a large extent, can be explained by the difficulties involved in the collection of quality data. Most often, the recognition accorded to emigration as one of the demographic processes, has not gone beyond the acknowledgement of its rightful position in the population equation as an algebraic component. Regardless of the attempts made in different circles to establish its association with brain drain, it still remains one of the least researched demographic topics. The purpose of this paper is to show that the impact of emigration on the countries of origin can be studied not only in terms of its lowering the quality and the rate of growth of the population, but in terms of other interesting and relevant aspects as well. It is hoped that some of these findings may encourage the national planners to initiate necessary measures for improving the coverage and quality of emigration statistics.

We begin our discourse in this paper by noting the similarity between emigration and death. Both are forces that are responsible for population decline and to that extent they can be treated in the same manner. Therefore, like the force of mortality μ (x) which is a measure of the pattern of depletion in the population by death specific for age x we can define a similar measure for emigration also spscific for age and call it the force of outward mobility and denote it by say $o(x)$. As is well known, the relation between $\mu(x)$ and the life table survivorship function $l(x)$ is by definition given by

$$-\frac{1}{l(x)}\frac{dl(x)}{dx} = \mu(x) \tag{1}$$

where the force of mortality is the only source of attrition. Therefore, in the presence of emigration, an expression parallel to (1) can be written as

$$-\frac{1}{l^*(x)}\frac{dl^*(x)}{dx} \quad \mu(x) + o(x) \tag{2}$$

where $l^*(x)$ is the proportion of the birth cohort alive at age x in the country of birth. For reasons of simplicity, we assume that there is no return migration. Next, from (1) and (2) we derive

$$l(x) = e^{-\int_0^x \mu(y)\,dy} \tag{3}$$

and

$$l^*(x) = e^{-\int_0^x \{\mu(y) + o(y)\}\,dy} \tag{4}$$

respectively. Due to (3), we can rewrite (4) as

$$l^*(x) = l(x)\, e^{-\int_0^x o(y)\,dy} \tag{5}$$

The *effect* of emigration on a cohort of babies as they grow old in their country of birth can be studied from (5). This has been attempted next where we have taken into consideration two simple models of emigration. In the first, the force of outward mobility has been taken as constant for all ages, i.e., $o(y) = k$. In the second, $o(y)$ has been assumed as proportional to the size of the cohort at that age, namely, $l^*(y)$ such that $o(y)$ — $c\, l^*(y)$ where c is a constant.

Special Case : $o(y)$ is a Constant

A. Measures Based on Mortality and Emigration

Substitution of $o(y) = k$ in (5) and subsequent simplification produces

$$l^*(x) = e^{-kx}\, l(x) \tag{6}$$

which shows that the ratio of $l^*(x)$ and $l(x)$ decreases exponentially with age. The nature of this decline over the entire range can be studied by computing the summary measure

$$e^*(x) = \int_0^{\alpha - x} l^*(x + t)\, dt = \int_0^{\alpha - x} e^{-k(x+t)}\, l(x + t)\, dt \tag{7}$$

which is the number of years that a person aged x can expect to spend in his country of birth and comparing it with $e(x)$, the standard measure of the expectation of life at age x. Naturally, the value of $e^*(x)$ at $x = 0$ provides the most comprehensive measure of the effect of emigration on a person's length of stay in the country of birth. This quantity, namely,

$$e^*(0) = \int_0^{\alpha} e^{-kx}\, l(x)\, dx \tag{8}$$

can be alternatively expressed as

$$e^*(0) = e^{-k\bar{x}} \int_0^{\alpha} e^{-k(\bar{x} - x)}\, l(x)\, dx \tag{9}$$

where $\bar{x}$ is the average age of the stationary population given by

$$\bar{x} = \frac{\int_0^{\alpha} x l(x) dx}{\int_0^{\alpha} l(x) dx} = \frac{\int_0^{\alpha} x l(x)\, dx}{e(0)} \tag{10}$$

and $e(0)$ is the expectation of life at birth. Expanding the exponential inside of the integral of (9) and simplifying we get

$$e^*(0) = e^{k\bar{x}} e(0) \left[1 + \frac{k^2}{2!} V(x) - \frac{k^3}{3!} M_3(x) + \dots \right] \tag{11}$$

where $V(x)$ is the variance and $M_3(x)$ is the third moment around the mean. Incidentally, an expression parallel to $\bar{x}$, namely,

$$\bar{x}^* = \frac{\int_0^\alpha x l^*(x)}{e^*(0)} = \frac{\int_0^\alpha x e^{-kx} l(x)dx}{\int_0^\alpha e^{-kp} l(x)dh} \tag{12}$$

is quite interesting. Observe that the integral in the numerator of (12) can be obtained by differentiating (8) with respect to $-k$. Therefore, its value should be the same as the derivative of (11) with respect to $-k$. Substitution of these results in (12) gives after appropriate simplifications

$$\bar{x}^* = \bar{x} - \frac{kV(x) - k^2 M_3(x) / 2! + \ldots}{1 + k^2 V(x) / 2! - \ldots} \tag{13}$$

Usually, k is quite small as the emigration rates rarely exceed three or four per thousand population. In that case, ignoring terms of the order of k^2 and above. we can write

$$e^*(0) = e^{-k\bar{x}} e(0) = (1 - k\bar{x})\, e(0) \tag{14}$$

and

$$\bar{x}^* = \bar{x} - kV(x) \tag{15}$$

showing that both $e^*(0)$ and $\bar{x}^*$ are smaller than their counterparts in the stationary population. It should be noted that more accurate values of these parameters can be obtained by first computing $l^*(x)$ from (6) and then calculating $e^*(0)$ and $\bar{x}^*$ from their respective standard formulas. For small k, however, (14) and (15) can be accepted as reasonable approximations.

The proportion of the cohort that emigrates and dies abroad is another interesting index which can be derived from $l^*(x)$ and $o(x)$ as (see eqn. 2)

$$p(e) = \int_0^\alpha l^*(x) o(x) dx = k \int_0^\alpha l^*(x)dx = ke^*(0) \tag{16}$$

as $o(x) = k$ for this special example. Therefore the complement $1 - ke^*(0)$ is the proportion that stays and dies in the country of birth.

The average age of emigrants at the time of emigration can be obtained as

$$\bar{x}(e) = \int_0^{\alpha} x\, l^*(x) o(x) dx \Big/ \int_0^{\alpha} l^*(x) o(x) dx \tag{17}$$

which after substitutions of the expression for $l^*(x)$ from (6) and of k for $o(x)$ reduces to

$$\bar{x}(e) = \frac{\int_0^{\alpha} x\, e^{-kx}\, l(x) dx}{\int_0^{\alpha} e^{-kx}\, l(x) dx}$$

which is the same as (12). That is to say,

$$\bar{x}(e) = \bar{x}^* \tag{18}$$

or that the average age of the emigrants is the same as that of the resident population.

The average number of years the emigrants would have lived if they had not left their country of birth is the sum of $\bar{x}(e)$ and

$$\bar{x}(o) = \frac{\int_0^{\alpha} k l^*(x)\, e(x) dx}{\int_0^{\alpha} k l^*(x) dx} \tag{19}$$

as those leaving the country at age x would have on an average an additional $e(x)$ years to live. Now, the numerator of (19) can be written as

$$\int_0^{\alpha} k e^{-kx}\, T(x) dx \tag{20}$$

where

$$T(x) = l(x) e(x) = \int_x^{\alpha} l(y) dy \tag{21}$$

Integration by parts reduces (20) to

$$T(0) - e^*(0) = e(0) - e^*(0) \tag{22}$$

substitution of which in (19) produces

$$\bar{x}(o) = \frac{e(0) - e^*(0)}{ke^*(0)} \tag{23}$$

Again, substitution of (14) simplifies (23) as

$$\bar{x}(o) = \bar{x} \tag{24}$$

as a first approximation. Thus an emigrant leaves his country of birth at an average age of $\bar{x}^*$ with $\bar{x}$ more years remaining to be lived. So their average age of death $e_m(0)$ can be expressed in terms of the sum of those two components as $\bar{x}^* + \bar{x}$. Naturally, the average age of death of those who survived the hazards of death until they emigrated at their respective ages must be greater than the overall life expectancy. That is to say

$$e(0) < e_m(0) \tag{25}$$

The average age of death of the stayers $e_s(0)$ can be similarly derived. Note that $e_s(0)$ is not the same as $e^*(0)$ since the former takes into account only those who died in their country of birth whereas the latter includes the contributions of not only the former but also of those who eventually left. As the number of stayers dying at age x is $l^*(x)\mu(x)$, the required age is given by

$$e_s(0) = \frac{\int_0^\alpha x l^*(x)\,\mu(x)\,dx}{\int_0^\alpha l^*(x)\,\mu(x)\,dx} \tag{26}$$

Substitution of (6) for $l^*(x)$ in (26), integration by parts and subsequent simplification produces

$$e_s(0) = \frac{e^*(0)(1 - k\bar{x}^*)}{1 - ke^*(0)} \tag{27}$$

Observe that the denominator of (27) is the proportion of the initial cohort that dies in the country of birth which is the same as the complement of (16) representing the stayers.

In most examples, $\bar{x}^* < e^*(0)$ and therefore, $e^*(0) < e_s(0)$ due to (27). Further we can also say that since the overall life expectancy $e(0)$ is a weighted average of the same for the stayers and the movers or of e_s (0) and e_m (0) respectively, e_s (0) has to be less than e (0) because of (25). Combining all of these we can therefore write

$$e^*(0) < e_s(0) < e(0) < e_m(0) \tag{28}$$

B. Measures with Fertility Included

The demographic process involving both mortality and fertility can be similarly analyzed for this model. We begin straightaway with the measure that is parallel to the net reproduction rate

$$R = \int_0^{\alpha} l(x)\, m(x)\, dx \tag{29}$$

which for this example can be written as

$$R^* = \int_0^{\alpha} l^*(x)\, m(x)\, dx = \int_0^{\alpha} e^{k-x}\, l(x)\, m(x)\, dx \tag{30}$$

Similarly, the corresponding average ages of motherhood are given by

$$a = \int_0^{\alpha} x l(x)\, m(x)\, dx / R \tag{31}$$

and

$$a^* = \int_0^{\alpha} x l^*(x)\, m(x)\, dx / R^* \tag{32}$$

The relationships between these two sets can be obtained in the same way that produced (14) and (15). These might be written as

$$R^* = R(1 - ka) \tag{33}$$

and

$$a^* = a - kV_m \tag{34}$$

where V_m is the variance of the distribution of the net maternity rates. Of course, as mentioned before, the values of these parameters can be directly obtained for any given value of k. However, the formulas expressing the relationships between these and their counterparts in the closed population have several advantages. Not only do they provide a comparison between the two sets but the formulas also provide quick estimates of the parameters for alternative values of k without going through all the steps again, beginning with the computation of $l^*(x)$. This is specially important for small k for which the approximations are quite good.

The implications of the above findings can be studied for any life table and for any given value of k. For a demonstration, we have chosen the 1985 life table of the Canadian females for which the following are given

$$e(0) = 79.69, \quad \bar{x} = 41.30, \quad R = .7923 \text{ and } a = 27.41$$

For $k = .003$ or an emigration rate of three per thousand population at all ages, the corresponding parameters turn out to be

$$e^*(0) = 70.71, \quad \bar{x}^* = 39.47, \quad R^* = .7309 \text{ and } a^* = 27.34$$

Thus, of the total life expectancy of 79.69 years, 8.98 years or more than 11 per cent will not be spent in the country of birth for an emigration rate of three per thousand. Also the proportion of the cohort emigrating is as large as 21 per cent (see 16) with the average age at the time of emigration as 39.47 years. Further, the emigrants would have lived an additional 42.34 years if they had not left giving rise to an $e_m(0)$ of 81.80 years. This value is larger than the overall $e(0)$ of 79.69 years which in turn is greater than the same for the stayers namely $e_s(0)$ which in this example is 79.11 years (see 28).

Special Case : $o(y)$ is Proportional to $l^*(y)$

In this model, we regard the size of the cohort at any age remaining in the country of birth as the sole determinant of mobility. The simplest relationship between $o(y)$ and $l^*(y)$ manifesting such a feature is given by

$$o(y) = cl^*(y) \tag{35}$$

where c is a constant. Substitution of (35) in (5) and subsequent simplification gives rise to

$$l^*(x) = l(x)\, e^{-c\{T^*(0) - T^*(x)\}} \tag{36}$$

In order to establish the functional relationship that expresses the l^* function with emigration in terms of the same without emigration, we first write (36) as

$$l^*(x)\, e^{c\{T^*(0) - T^*(x)\}} = l(x) \tag{37}$$

from which we can write the integral equation

$$e^{cT^*(0)} \int_x^{\alpha} e^{-cT^*(y)} l^*(y)\, dy = \int_x^{\alpha} l(y)\, dy \tag{38}$$

Since $dT(y)/dy = -l(y)$ (see 21), (38) simplifies into

$$\frac{e^{cT^*(0)}}{c} \{1 - e^{-cT^*(x)}\} = T(x) \tag{39}$$

For the special case of $x = 0$, we get from (39)

$$\frac{e^{ce^*(0)} - 1}{c} = e(0) \tag{40}$$

Since $T^*(0) = e^*(0)$. Taking logarithms, (40) becomes

$$ce^*(0) = \{ln(1 + ce(0)\} \tag{41}$$

When $ce^*(0) < 1$, (41) can be written as

$$e^*(0) = e(0) \left(1 - \frac{ce(0)}{2} + \cdots \right) \tag{42}$$

Now we can rewrite (39) as

$$e^{c\{T^*(0) - T^*(x)\}} = 1 + cT(0) - cT(x) \tag{43}$$

by using (40) and rearranging terms. Next, substitution of (43) in (35) provides the desired explicit relationship between $l^*(x)$ and $l(x)$ as

$$l^*(x) = \frac{\partial(x)}{1 + cT(0) - cT(x)} \tag{44}$$

Once the $l^*(x)$ values are determined from a life table for a specific value of c we can derive the values of all the other parameters associated with the distribution of $l^*(x)$. For example, the number of emigrants can be obtained from

$$\int_0^{\alpha} l^*(x)\, o(x)\, dx = c \int_0^{*} \{l^*(x)\}^2\, dx = c \int_0^{\alpha} \frac{l^2(x)\, dx}{\{1 + c\, T(0) - cT(x)\}^2} \tag{45}$$

In terms of a cohort of size one, (45) represents the proportion of the cohort *p (e)* that will emigrate over the entire length of life. The algebraic expressions for any other parameter can be similarly derived.

Using once again the Canadian example with an *e* (0) of 79.69 years, an arbitrary value of *c* = .003 produces the corresponding value *of e** 0) as 71.4 years as against 70.71 years for the same in our first model. A comparison between (16) and (45) reveals that when all other things remain the same identical values of *k* and *c* will produce larger values of *e** (0) for the first model. In turn it is expected to produce somewhat smaller value of *p* (e) for the second model. This indeed is the case as its value is about 17 per cenl for the second model as against 21 per cent for the first.

Summary and Concluding Remarks

The two models of emigration discussed in the foregoing were chosen for the analytical advantages they offer in determining their effects on other demographic measures. At the same time the patterns of emigration depicted by them seem to be quite realistic. It is true that from the point of view of mathematical simplicity alone any form of *o (x)* that could express /* *(x)* as a function of / *(x)* and other functions related to / *(x)* would be highly desira.ble. Few of these functions may, however, be meaningful in terms of their ability to describe the pattern of emigration to a reasonable degree of approximation.

As an example, consider the case in which *o (x)* is regarded as proportional to (* *(x),* i.e., *o (x)* = *hμ. (x)* where *h* is a constant. The solution of (5) can then be obtained as

$$l^* (x) = [l (x)]^{1+h} \qquad (46)$$

But (46) appears to be more appropriate for modeling mortality change rather than emigration.

Be that as it may, the important point to note is that the model, in its general form (see eqn. 5), is perfectly amenable to appropriate adjustments that are necessary to deal with any specific form of *o (x).* In its ultimate crudity, the procedure is similar to that of deriving a life table from any set of age-specific mortality rates where these rates are used as substitute measures of the force of mortality at the centers of the respective age intervals. In the same way, the rates of emigration specific for age can serve as measures of the force of outward mobility. The exponent in (5) can thereafter be obtained simply by cumulating those rates with appropriate adjustments for the length of the age intervals.

Simply put, the effect of emigration on a cohort is cumulative in nature.

This may be seen from (5) which can be rewritten as

$$l^* (x) = u (x)\, l (x) \qquad (47)$$

where u(x), is a monotonic nonincreasing function of x such that $0 < u (x) < 1$. Also

$$u' (x) = -u (x)\, o (x) \qquad (48)$$

The net effect of $u (x)$ on $l(x)$ is that the ratio of $l^* (x)/l(x)$ gets smaller and smaller as x increases. As a result, the proportion of the expected length of life lived in the country of birth gets smaller than 90 per cent when emigration rate is held constant at .003 at all ages.

At this point we would like to mention the negative effect that emigration has on the expectation of life at birth which appears to have gone unnoticed in demographic literature. The simple mathematical fact is that the life expectancy of a closed population is greater than that of the stayers (in a population subjected to emigration). This is because those who emigrate at age x would die at an average age of $x + e (x)$ years if they stayed. As is well known $x + e (x)$ increases with age with a minimum value of e (0) at age 0. In order to balance, the average age of death of the stayers has to be smaller than e (0) which in the example discussed in the paper turned out to be smaller by about seven months. In the same way, the measure parallel to the net reproduction rate is an underestimate of what it would have been in a closed population (see eqn. 31). In the example selected for discussion the reduction in net reproduction rate was almost 8 per cent. In summary it can be said that emigration is not just an independent component of population change with no effect on measures based on mortality and fertility. Among others, lowering of life expectancy and the net reproduction rate are two of the major consequences of emigration. It is hoped that future investigation will shed more light on significant effects of this component on other demographic measures.