Demography India Vol. 28. No. 2 (1999). pp. 153-165

P. N. Mari Bhat*

Population Projections for Delhi: Dynamic Logistic Model Versus Cohort-Component Method

DEMOGRAPHERS are often asked to make population projections for small areas, such as for a city. The available techniques to do so are limited, and little is known about their track record. The standard cohort-component procedure is often found wanting, as it is not easy to predict the flow of migrants in and out of an area. Several years ago, I had the humbling experience of finding my estimates way off the mark when I attempted a population projection for Bangalore City (cited in Rao and Tiwari, 1979). Recently my assistance was sought to make population projection for the National Capital Territory of Delhi, for the Master Plan being prepared by the Delhi Development Authority for 2021. One of the methods used in this exercise was the 'dynamic' or variable carrying capacity logistic model, a technique perhaps not many demographers are familiar with. Since this method may have wider applicability in demographic analysis, it is being discussed here in detail, and its forecasts of population are compared with those of the cohort-component method.

Limitation of the Logistic Model

The logistic curve was being widely used in population projections before the cohort component method became popular (e.g., Davis, 1951). Generally, predictions from the logistic model have turned out to be gross underestimates. A major reason for its poor performance is that population growth rate had not began to fall when attempts were made to fit the model. Under the logistic growth, the annual increase in population first rises before it falls, while the growth rate of population falls continuously with time. This is evident by examining the differential equation of the logistic curve:

* Professor and Head, Population Research Centre, Institute of Economic Growth, Delhi-110 007.

$$\frac{dP_t}{dt} = bP_t\,(K-P_t)$$

$$\frac{dP_t}{P_t\,dt} = b\,(K-P_t) \qquad (1)$$

where K is the upper ceiling on population size, and b is a constant.
Equation (1) basically states that under the logistic curve, the annual *percentage* increase in the population is a constant proportion of the difference between current population and the upper limit to the population size. Since as P_t increases the potential for further growth, $K - P_t$ falls, the growth rate of population under the logistic curve would fall continuously with time from the very beginning itself. This is not to be confused with the fact that the annual increase in population (dP_t/dt) under the logistic curve has a symmetric, bell-shaped distribution, as indicated by the first part of eq. (1).

Rather than being subjected to a regime of continuously falling growth rates, most human populations have a phase of rising growth rates followed by a period of declining growth rates. The data shown in Table 1 for Delhi is a case in point. The growth rate of Delhi's population rose from 0.2 percent during 1901-11 to 3.7 percent during 1931-41. Since then, except for an unusually high growth rate during 1941 -51, the growth rate of Delhi's population has remained more or less stationery at around 4.0 percent per annum. The unusually high growth rate during 1941 -51 was due to the partition-related influx of migrants from Pakistan. If a correction were made to this atypical event, the growth rate for this period too would be under four percent. Therefore, rather than the logistic, an exponential curve would fit better to the recent history of population growth in Delhi.

Dynamic Logistic Model

Because of likely constraints on carrying capacity of land, fitting a ^-shaped curve remains an attractive option while making long-term projections of city sizes. The standard logistic curve provides poor fit to past population trends mainly because of its underlying assumption that the ceiling on population size is time invariant. Owing to technological progress, capacity of the land to accommodate population might be rising, and this could explain why growth rates tend to rise, or remain stationary, when a fall is expected under the standard model. This limitation of the logistic curve can be remedied by assuming the upper asymptote vary with time:

$$\frac{dP_t}{P_t\,dt} = b\,(K_t-P_t) \qquad (2)$$

Although applications of this 'dynamic' logistic model (or, time-dependent carrying capacity model) have been rare in formal demography, it has been used extensively in studying diffusion processes (see Mahajan and Peterson, 1985; Banks, 1994). In actual estimation of this model, some arbitrariness cannot be avoided, as a functional form must be imposed for *K*,. While *K*, could take many forms, in practice a function involving more than two parameters may prove difficult to handle. The two simple forms discussed below gave plausible results in the case of Delhi.

Case I: Linear Change in Upper Asymptote

A simple linear form for *K*, is the easiest to estimate. In this case it is assumed that

$$Kt = Ko + kt$$

With this assumed form for *Kt*, eq. (2) can be rewritten as

$$\frac{dP_t}{P_t\,dt} = bK_0 + bkt - bPt \qquad (3)$$

The discrete analogue of eq. (3) can be written as

$$\text{In } P_{t+1} - \text{In } P_1 = bK_0 + bkt - bP_t \qquad (4)$$

Eq. (4) can be estimated through linear regression of the difference in the logarithm of population sizes at two successive censuses on time and population in the first census. If *A* is the intercept, *B* and *C* are the estimated coefficients of time and population respectively, then

$$b = -C; \quad K_0 = -A/C; \quad k = -B/C.$$

It is to be noted that the model expects the coefficient *of Pt* (i.e., C) is < 0, and that of time *t* (i.e., *B) to* be > 0. But since both *t* and *Pt* can be highly correlated, one may not always get meaningful results if the model is fitted to a short time series.

Once the model parameters are estimated, population for any time can be derived directly from the first difference equation:

$$\text{In } P_{t+1} = \text{In } Pt + bK_0 + bkt - bPt \qquad (5)$$

Case II: Relational Upper Asymptote

A problem with the linear form for the upper asymptote is that it does not indicate any finite upper limit to city size. There could be limits to what technological advances can achieve, and it may not be possible to accommodate ever-increasing numbers in a small area. An alternative is to assume that the ceiling on city's population is a constant proportion of the population of a larger area that the city is part of. That is,

$$Kt = aNt,$$

where N_t *is* population of the country, or the state, or the district where the city is located. In effect, it is assumed that the city size would grow asymptotically reach αN_u, where N_u is the finite upper limit to N_t With this assumed form, eq. (2) can be rewritten as

$$\frac{d P_t}{P_t\, dt} = abN_t - bp_t \qquad (6)$$

For estimation, we may use the following discrete analogue of eq. (6):

$$\text{In } P_{t+1} - \text{In } P_t = \alpha b N_t - bP_t \qquad (7)$$

As before, eq. (7) can be estimated through linear regression, but without the intercept term. If B and C are the estimated coefficients *of* N_t and P_t respectively, then

$$b = -C; \text{ and } a = -B/C$$

However, to forecast population size of the city, it would be necessary to have at one's disposal population projections for the state or the country where the city is located (i.e., N_t). If such estimates are available, population of the city for any time can be derived directly from the first difference equation:

$$\ln P_{t+1} = \ln P_t + \alpha b N_t - bP_t \qquad (8)$$

Application to NCT of Delhi

First, the dynamic logistic model was fitted to the population data of NCT of Delhi assuming linear form for the upper asymptote. The data for the entire time range, 1901 to 1991 were used, after adjusting the growth rate of 1941 -51 for refugee migration from Pakistan. The resulting equation was:

$$r_t, = 0.8922 + 0.0843 \text{ t} - 0.0586\, P, \qquad R^2 = 0.922;\ N = 9$$

$$(2.92) \quad (6.46) \quad (3.32)$$

where r, is the intercensal exponential growth rate, with / as the transformed calendar year with the origin at 1901. The figures in parentheses show the t-statistic of the estimated model parameters.

As indicated by the R^2 of 0.92, the model fits very well to the data. As anticipated, the estimated coefficient of time is positive while that of population is negative. The estimated equation implies that the ceiling on Delhi's population was growing linearly from 15.23 lakh in 1901, with a constant annual increase of 0.0843/0.0586 = 1.44 lakh. That is

$$Kt = 15.23 + 1.44t$$

To make population estimates, it is needed only to input the 1901 population figure for Delhi. All other population estimates, for the observation period and beyond, could be generated from the growth rates derived from the estimated equation.' The model population estimate for 1991 (93.9 lakh) is very close to the observed figure (94.2 lakh). The model forecasts a population of 12.6 million for 2001 and 17.3 million for 2021 (see Table 1). More importantly, as the estimated growth rates show, the dynamic model can accommodate both rising and falling growth rate regimes. However, as per this model, Delhi's population would never cease to grow, but would close in on its dynamic upper limit (see Figure 1). In 2051 the difference between the two would be only 11 lakh (231 versus 220 lakh).

The second version of the model was fitted to the same data by assuming that upper asymptote was a constant proportion of all-India population. In this case population data for 1921 to 1991 were used, since the model fitted better to this time range. The least-squares regression with no constant term resulted in the following equation:

$$r_t = 0.0139\, N_t - 0.8263\, P_t \qquad R^2 = 0.526;\ N = 7$$

(11.15) (4.69)

As the R^2 value of 0.526 shows, the model fit was poorer than the linear version of the model.[2] However, the fit to past data series may not be a good indicator of performance of the model during the forecasting phase. As per the fitted model, the Delhi's population would grow asymptotically to reach 0.0139/0.8263 = 1.68 percent of India's population.

To derive population forecasts from this model, it is necessary to have population projections for all-India for the corresponding years. Here we have used estimates derived from a cohort-component method that assumed that total fertility at the all-India level would attain the replacement level (i.e., 2.1 births per woman) during 2016-21, and would remain at that level during the projection period. According to this projection, India's population would stabilise at about 1.8 billion. This has the implication that upper limit to Delhi's population is 1,800 x 0.0168 = 30 million.

The model forecasts of Delhi's population for any year can be derived by plugging in eq. (2) the projected figures for all-India. The resulting estimates of population of Delhi are 21 million in 2021 and 27 million for 2051. Thus compared to the linear version of the model, this version anticipates 4 million more population in 2021 and 5 million more in 2051. Delhi's population would be pretty close to its upper limit of 30 million by 2051 (see Figure 1).

---

[1]However, for 1951 the number of immigrants to Delhi would have to be added as the adjusted growth rate had been used in model estimation.

[2]*To* achieve the comparison, we have shown the R^2 derived from the total sum of squares computed from deviations from the mean of the dependent variable. However, in regressions with no constant term no such 'mean corrections' are generally made. The R^2 computed without the mean correction is 0.988, indicating an excellent fit.

TABLE 1: PROJECTING DELHI'S POPULATION THROUGH DYNAMIC LOGISTIC MODEL

Year	Re-scaled time (t)	Population in million: Delhi (Pt)	Population in million: All-India (Nt)**	Intercensal exponential growth rate of Delhi (r_t)	Linear change in upper limit: Upper limit to population	Linear change in upper limit: Population growth rate	Linear change in upper limit: Projected population @ (in million)	Constant ratio upper limit: Upper limit to population	Constant ratio upper limit: Population growth rate	Constant ratio upper limit: Projected population @
1	2	3	4	5	6	7	8	9	10	11
1901	0	0.406	238.4	0.20	1.52	0.65	0.41			
1911	10	0.414	252.1	1.66	2.96	1.48	0.43			
1921	20	0.488	251.3	2.64	4.40	2.28	0.50	4.23	3.09	0.49
1931	30	0.636	279.0	3.67	5.84	3.05	0.63	4.70	3.36	0.67
1941	40	0.918	318.7	6.42 (3.08)*	7.28	3.76	0.86	5.37	3.68	0.93
1951	50	1.744	361.1	4.22	8.72	4.09	1.74	6.08	3.58	1.84
1961	60	2.659	439.2	4.25	10.16	4.41	2.62	7.40	3.91	2.63
1971	70	4.066	548.2	4.25	11.60	4.40	4.08	9.23	4.27	3.89
1981	80	6.220	683.3	4.15	13.04	3.93	6.34	11.51	4.37	5.97
1991	90	9.421	846.3		14.48	2.98	9.39	14.25	3.99	9.24
2001	100		1019.8		15.92	1.92	12.65	17.17	2.81	13.77
2011	110		1189.9		17.36	1.19	15.32	20.04	1.49	18.24
2021	120		1331.6		18.80	0.90	17.26	22.42	1.04	21.16
2031	130		1460.2		20.24	0.79	18.89	24.59	0.91	23.49
2041	140		1567.4		21.68	0.72	20.44	26.39	0.55	25.72
2051	150		1647.2		23.12	0.67	21.98	27.74	0.45	27:19

** Figures for 2001 onwards were derived using the component method.

* Adjusted by subtracting 495 thousand immigrants from Pakistan.

@ 495 thousand immigrants from Pakistan were added to the population estimate for 1951.

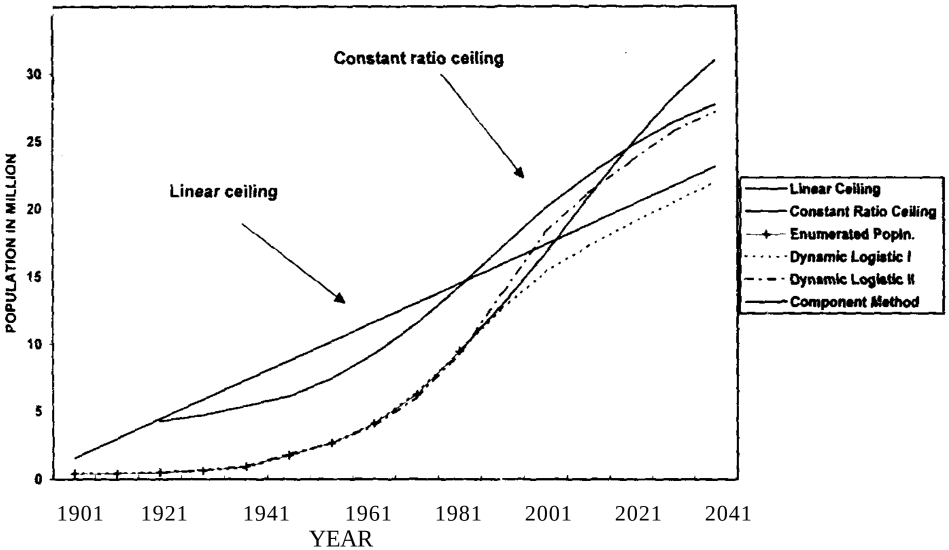

Fig. 1. Population Estimates for Delhi, 1901-2051: Forecasts from Dynamic Logistic & Component Methods

Projection by Cohort-Component Method

It is useful to test the forecasts of Delhi's population from the dynamic logistic model with those from a cohort-component method, or vice-a-versa. A cohort-component projection for Delhi was made in the following manner. The age-sex distribution from the 1991 census was first smoothed using a moving-average formula, and by assuming that proportion of population below the age 15 years has been correctly enumerated in the census. We then projected forward the 1991 population to 1996 by using the fertility rates derived from the data of the National Family Health Survey, 1992-93, and survival rates from the life table for 1989-93 for urban areas of Haryana from the Sample Registration System. The crude birth and death rates for 1991-95 implied by this projection were compared with the SRS estimates for Delhi for the same period. The assumed age-specific fertility rates and survival odds were proportionately adjusted such that they would replicate the levels of crude birth and death rates of the SRS for Delhi during 1991-95. In this manner, the total fertility rate in Delhi during 1991-95 was estimated to be 2.9 births per woman, and life expectancy at birth for males and females as 68 and 71 years, respectively.

The level of TFR was assumed to fall linearly from 2.9 births per woman to 2.1 in 2001-05. The preliminary results from the second round of the NFHS (1999), and RCH survey of 1998 indicate that decline of fertility of this magnitude has been occurring in Delhi. The trends in SRS crude birth rates during the 1990s also confirm this.

According to the SRS, the crude birth rate has fallen from 24-25 in early 1990s to 21 per 1000 in 1997. There are also indications from other parts of India that the downward trend in fertility does not come to a halt after reaching the replacement level. Given the strong preference for sons in northern India, it is conjectured that TFR may fall up to a level of 1.5 birth per woman, if most couples stop childbearing after having one son. Under this stopping rule, with some allowance made for sterility, the distribution of completed family size with a mean of 1.5 would resemble the following:

Completed Family Size:	0	1	2	3+	Mean
Percent women:	10	45	30	15	1.5

It was, therefore, assumed that TFR in Delhi would gradually fall up to 1.5 births per woman and stay at that level. Although fertility can fall further if couples cease to have preference for sons, for the present, it does not seem a realistic option.

With the threat of AIDS looming large, overcrowding and pollution levels on the rise, a conservative assumption was adopted in projecting mortality levels. The expectation of life at birth among males was assumed to rise by only one year in every 10 years. Among females, the rise was assumed to be by one year in every five years until the life expectancy at birth reaches 74 years; thereafter the rise was assumed to be at the same pace as among males.

However, the most crucial assumption is regarding migration flows to Delhi. The inspection of data from the past censuses suggests that net migration to Delhi has increased almost linearly from 0.63 million during 1961-71 to 0.95 million during 1971-81 and to 1.4 million during 1981-91. The majority of migrants during the last decade came from Uttar Pradesh (55%) and Bihar (13%). It was, therefore, assumed that migration to Delhi would grow at the same rate as the population in the age group 15-29 in Uttar Pradesh and Bihar. Separate projections made for Uttar Pradesh and Bihar provided the required data on the expected growth of population in the relevant age group during 1991 to 2051. By applying this growth pattern to migrants to Delhi, the size of migration flows was projected to the future. Table 2 shows the projected figures of net migrants, as well as the TFR and life expectancy levels.

The projection needed age-specific values of fertility, mortality and migration rates. The age-specific fertility rates during the projection period were derived by assuming that the decline would be higher in the age group 15-19 years and 30 years and over. The age-specific mortality rates were derived by assuming that the estimated values for 1991-95 would linearly coverage to those of the West Model life tables of level 27. For allocating estimated total migrants to various age and sex categories, a standard distribution was assumed. This standard was derived from the a,ll-India data on rural-urban migration flows during 1981-91.

Some important results of population projection made using these assumptions are reported in Table 3. According to the component projection, Delhi's population would grow from 9.4 million to 20.8 million in 2021. Population forecasts from this method

TABLE 2: KEY ASSUMPTIONS AND UNDERLYING DEMOGRAPHIC RATES IN PROJECTION OF DELHI'S POPULATION BY COMPONENT METHOD

Period	*Life expectancy at birth*		*Total fertility rate*	*Net migrants (000s)*	*Rates per Net 1,000 population*			
	Males	*Females*			*Net migration*	*CBR*	*CDR*	*Natural increase*
1991-95	68.0	71.0	2.9	759	14.8	24.3	5.8	18.4
1996-00	68.5	72.0	2.5	842	14.0	21.5	5.7	15.8
2001-05	69.0	73.0	2.1	940	13.6	18.7	5.6	13.1
2006-10	69.5	74.0	1.9	1055	13.4	17.2	5.8	11.4
2011-15	70.0	74.5	1.8	1166	13.2	16.0	6.0	10.0
2016-20	70.5	75.0	1.7	1246	12.6	14.6	6.3	8.3
2021-25	71.0	75.5	1.6	1293	11.8	13.2	6.6	6.6
2026-30	71.5	76.0	1.5	1318	11.1	11.9	7.1	4.8
2031-35	72.0	76.5	1.5	1311	10.2	11.5	7.6	3.9
2036-40	72.5	77.0	1.5	1269	9.3	11.1	8.2	3.0
2041-45	73.0	77.5	1.5	1204	8.3	10.7	8.7	2.0
2046-50	73.5	78.0	1.5	1204	7.9	10.1	9.2	0.9

TABLE 3: KEY RESULTS OF PROJECTION OF DELHI'S POPULATION BY COMPONENT METHOD

Year	*Population (in '000s)*	*Sex ratio (F/M)*	*Growth rate of population (%)*				*% of population at ages*			*Dependency ratio %*	
			All ages	*0-14*	*15-59*	*60+*	*0-14*	*15-59*	*60+*	*0-14*	*60+*
1991	9,421	827					34.8	60.4	4.8	57.6	7.9
1996	11,122	841	3.3	1.8	4.0	4.9	32.2	62.5	5.2	51.5	8.3
2001	12,912	853	3.0	1.2	3.6	5.5	29.5	64.6	5.9	45.7	9.1
2006	14,752	863	2.7	0.7	3.2	5.5	26.8	66.4	6.8	40.4	10.2
2011	16,703	872	2.5	0.9	2.8	5.2	24.7	67.5	7.8	36.6	11.6
2016	18,756	881	2.3	0.9	2.4	5.6	23.0	67.8	9.2	33.9	13.6
2021	20,825	888	2.1	0.9	2.0	5.1	21.7	67.6	10.7	32.1	15.8
2026	22,832	896	1.8	0.6	1.8	4.6	20.4	67.3	12.3	30.3	18.3
2031	24,719	903	1.6	0.1	1.6	3.9	18.9	67.2	13.8	28.1	20.5
2036	26,526	910	1.4	0.2	1.3	3.6	17.8	66.8	15.4	26.6	23.1
2041	28,199	917	1.2	0.4	1.0	3.2	17.1	65.9	17.0	25.9	25.8
2046	29,687	924	1.0	0.4	0.6	3.0	16.6	64.6	18.8	25.7	29.1
2051	31,028	931	0.9	0.3	0.5	2.6	16.1	63.4	20.5	25.4	32.3

fall in between those derived from the two versions of the dynamic logistic model. However, projected populations for years after 2021 are higher than those implied by the two versions of the dynamic logistic model. The component projection suggests that Delhi's population would cross 30 million in 2051, whereas the constant-ratio (relational) logistic model had put the figure at 27 million. Considering the possible range for uncertainty, these are relatively minor differences.

As an important by-product, the component projection also gives estimates of population by age and sex. It shows that there would be a steady increase in the sex ratio of population, from 827 females per 1000 males in 1991 to 888 in 2021 and 931 in 2051. This is a direct consequence of the widening gap between the life expectancies of the two sex, as the sex ratio at birth has been assumed to remain unchanged at 106 boys per 100 girls, and migration to Delhi has been assumed to be more masculine in nature (55 males for 45 females). The population estimates by age show that the age interval 60 years and over would be the fastest growing age segment with an average growth rate of over five percent per annum during 1991-2021. The old age population would continue to grow at a fast rate (over 2.5 percent) even up to 2051. Consequently, the percentage of population aged 60 and over would increase from 5 percent in 1991 to 11 percent in 2021 and 20 percent in 2051. By 2041, population aged 60 years and over would exceed the population at ages less than 15 years.

Although the dynamic logistic model cannot supply information on the nature of changes expected in the age-sex structure of the population, it can usefully provide some independent check on the projected total population from the component method. -We are, therefore, more confident in proposing the following ranges for Delhi's population for some selected dates in future:

Year	*Population*
2001	12.5 - 14 million
2006	14 - 16
2011	15 - 18
2016	16 - 20
2021	17 - 21
2051	22-31

It should however be noted that while making these projections no adjustment was made for the possible underenumeration of population in 1991. According to the post-enumeration check of the 1991 census, population of the three metropolitan cities, Delhi, Mumbai and Chennai together had a net undercount of four-percent (India, Registrar General, 1994). If such an undercount is assumed in the base population, all projected figures for Delhi presented in this paper would have to be raised by about four percent.

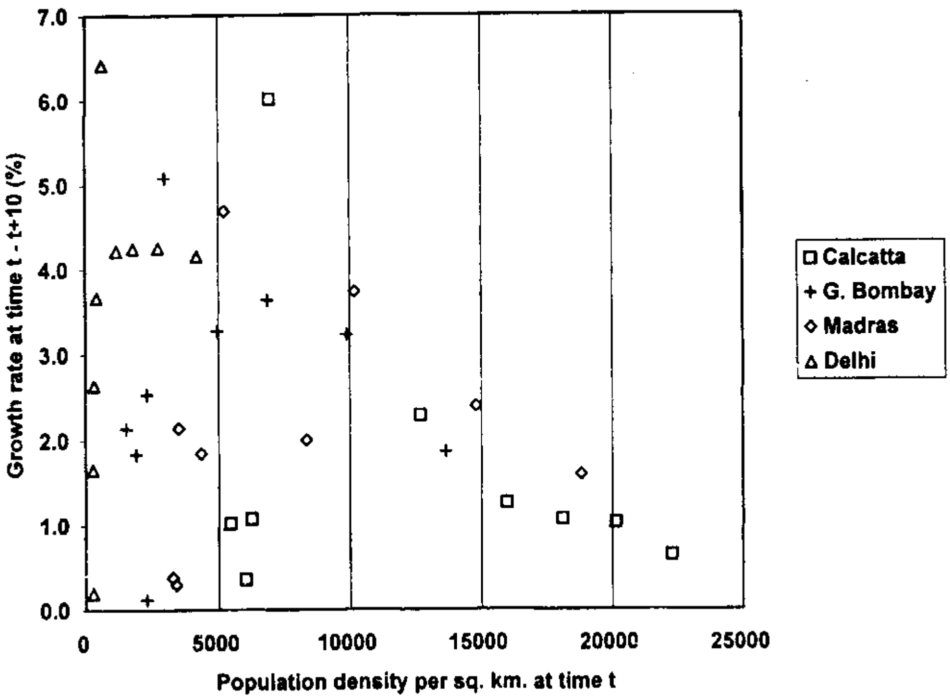

Fig. 2. Relationship between growth rates and population density, Calcutta, G. Bombay, Madras and Delhi, 1901 to 1991

Threshold Population Density for Indian Cities

One would naturally expect the growth rate to slow down as population reaches the carrying capacity of the area. Is there any evidence of this occurring in large Indian cities? If so, at what level of population density is it seen? Figure 2 shows the relationship between population density and growth rate of population as observed in the largest four metropolitan districts of India, namely, Calcutta, Greater Bombay, Chennai and Delhi, during the nine decades beginning from 1901-11. In all, there are 36 pairs of observations on population density (at the beginning of a decade) and growth rate (during the decade), 9 each for the 4 metropolitan districts. It is clear from the graph that widely varying population growth rates have been recorded at density levels under 10,000 persons per square kilometre (100 per hectare). However, after attaining a density of 10,000 persons per sq. km., no city has registered a rise in the growth rate. In fact, all of those who have achieved this critical density (i.e., Calcutta, Chennai and Bombay) have registered sharp falls in the growth rate in subsequent periods. Delhi with a population density of 6,352 per sq. km. in 1991 is expected to reach this mark during 2001-11. However, fall in the growth rate of Delhi is predicted even before reaching this mark. This may be because Delhi has the largest geographical area among the four cities. High concentration of population is possible in small geographical pockets having some special features; it may not be possible to sustain the same kind of pressure on resources in a larger land area.

Conclusion

Because of likely constraints on carrying capacity of land, fitting a S-shaped curve is a useful option while making long-term population projections for cities. However, as the carrying capacity is expected to rise with technological progress, better forecasts can be obtained by making the upper asymptote of the logistic curve a function of time. This paper discusses the methods of fitting such a 'dynamic' logistic model to population trends of Delhi, and compares the model forecasts with those of a cohort-component projection. The results indicate that Delhi's population is growing asymptotically to reach 1.7 percent of India's population, and the city size would triple before stabilisation is attained.

References

Banks, R. B., 1994, *Growth and Diffusion Phenomena: Mathematical Frameworks and Applications.* Berlin: Springer-Verlag.

Davis, K... 1951, *The Population of India and Pakistan*. Princeton NJ: Princeton University Press.

India. Registrar General, 1994, *Report on Post-Enumeration Check*. Census of India 1991, Series-1. Paper 1 of 1994. New Delhi: Office of the Registrar General.

Mahajan, V. and Peterson, R. A., 1985. *Models for Innovation Diffusion*. Sage University Paper Series on Quantitative Applications in the Social Sciences, No. 48. Newbury Park CA: Sage.

Rao, V. L. S. P. and Tiwari, V. K., 1979, *Structure of an Indian Metropolis: A Study ofBangalore*. New Delhi: Allied.