Demography India
Vol.J8(1989),pp.95-102

S. Biswas and Monthir A. Abid

On the Estimation of Mean Infecundable Period following Childbirth

Introduction

THE purpose of this paper is to estimate the mean period of natural infecundability following a childbirth. Assuming that the expiry of P.P. A. or lactation are caused by two forces which are competing with each other. Lactation period following a childbirth is considered to be infecundable. However, sometime, reconception of fresh menstrual cycle even before the natural discontinuance of lactation period brings a mother under the risk of reconception. Our object is to measure the expected longevity of the lactation period given that the expiry of P.P.A. takes place at sometime after the same. In this case lactation period may be taken as the measure of infecundable period unless expiry date of P.P.A. is substantially larger. On the other hand, if expiry of P.P. A. occurs before the lactation period, the natural infecundability period would be given by the mean period of P.P. A. given that expiry of lactation follows the same. Since the first and second measure may provide little under-estimation and overestimation respectively; therefore the two measures of mean infecundable period should be weighted by the respective probabilities of the events [viz. *P* (P.P.A. > lactation) or *P* (lactation > P.P.A.)] and the mean infecundable -period may therefore be estimated with precision. The present exercise is devoted to the same. Let *Xi* be a r. v. representing the period of Post Partum Amenorrhoea (P.P. A.) following a childbirth and *X2* be a r.v. representing the period of lactation following a childbirth.

The expiry of P.P.A. or the period of lactation eventually brings a mother further into the risk of conception. The forces causing the expiry of P.P. A. and that of lactation period may be considered as two risks (correlated) competing with each other to cause reconception. In this problem an attempt has been made to provide a methodological solution for estimating the mean length of the lactation period when P.P. A. is expired only after the same; and the mean length of the PP. A. when lactation period is still continued even with the cessation of the same. The entire exercise is methodologically illustrated by using Freund's (1961) bivariate exponential model representing the two variable which may be identified as *(Xi* = Period of P.P. A.), *(X2* - Period of lactation). Further, the suitability of the model representing

* *'P'* stands for probability.

the interaction between P.P.A. and lactation is given by the following features of the model viz:

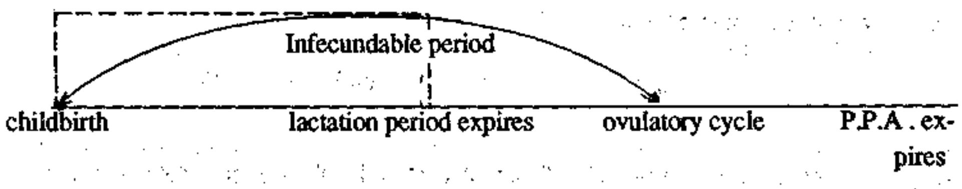
I. The model distinguishes the joint probability distribution of P.P.A. (X_1) and lactation period (X_2) according as $X_1 < X_2$ or $X_1 > X_2$.

II. Given that P.P.A. is over with intensity α and lactation with intensity β; given that on expiry of P.P.A.(X_1) with intensity α the expiry of lactation period would be with much great intensity β' rather than β ($\beta' \geq \beta$)

Similarly, given that the expiry of lactation period is caused first with intensity β, the expiry of P.P.A. would be accelerated with a much greater intensity α' rather than α ($\alpha' \geq \alpha$).

These features make Freund's model suitable (which is otherwise very suitable model to represent several problems is survival analysis and competing risk theory).

Estimate of the infecundable period.



Estimate of the infecundable period.

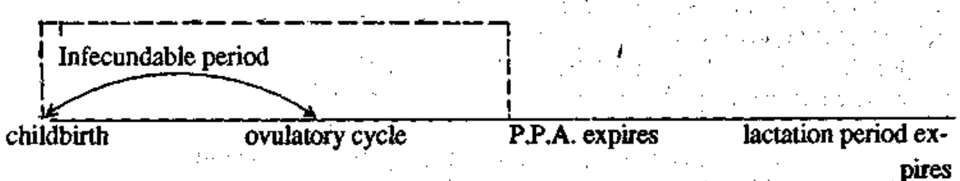


Figure 1.

Methodology

Let Freund's Bivariate exponential model represent the joint distribution of P.P.A. (X_1) and the lactation period (X_2) given by :

$$F(x_1, x_2) = \begin{cases} F_1(x_1, x_2) = \alpha\beta' \exp[-\beta' x_2 - (\alpha + \beta - \beta') x_1] \ ; 0 < x_1 < x_2 & (1a) \\ F_2(x_1, x_2) = \alpha'\beta \exp[-\alpha' x_1 - (\alpha + \beta - \alpha') x_2] \ ; 0 < x_2 < x_1 & (1b) \end{cases}$$

(1a) represents the case when P.P.A. preceds the lactation period, and (1b) represents the case when lactation preceds the P.P.A.

Then following the concept of competing risk theory (David and Moeschberger 1978) we define the death density functions of P.P.A. and lactation period by f_1 (.) and f_2 (.), where

$$\widetilde{f_1(x_1)} = \frac{P_1(x_1)}{\pi_1} \int_{x_1}^{\infty} P(x_2 \mid x_1)\, dx_2 \tag{2}$$

$$\widetilde{f_2(x_2)} = \frac{P_2(x_2)}{\pi_2} \int_{x_2}^{\infty} P(x_1 \mid x_2)\, dx_1 \tag{3}$$

Here $\widetilde{F_1(x_1)}$ represents the conditional probability of the length of P.P.A. lying between (x_1, $x_1 + \sigma x_1$) as $\sigma x_1 \to 0$ given that expiry of P.P.A. precedes the lactation period; f_2 (x_2) represents the conditional probability of the length of lactation period lying between (x_2, $x_2 + \sigma x_2$) as $\sigma x_2 \to 0$ given that the expiry of lactation period precedes the length of the P.P.A.; π_1 and π_2 represent the unconditional probabilities of P.P.A. lactation period and lactation period P.P.A. respectively; P_1 (x_1), $P_2(x_2)$ represents the unconditional probability of P.P.A. and lactation period lying between (x_1, $x_1 + \sigma x_1$) as $\sigma x_1 \to 0$ and (x_2, $x_2 + \sigma x_2$) as $\sigma x_2 \to 0$ respectively; and $P(x_2 \mid x_1)$ and P ($x_1 \mid x_2$) represent the conditional probability of lactation period lying between (x_2, $x_2 + \sigma x_2$) given the P.P.A. is equal to x_1, and the conditional probability of P.P.A. lying between (x_1, $x_1 + \sigma x_1$) given the lactation period is equal to x_2.

Then

$$\widetilde{E(X_1)} = E\,(X_1 \mid X_2 > X_1) = \int_0^{\infty} x_1 \widetilde{F_1\,(x_1)}\, dx_1 \tag{4}$$

= Mean length of the P.P.A. under the condition that lactation succeeds P.P.A.

$$\widetilde{E(X_2)} = E\,(X_2 \mid X_1 > X_2) = \int_0^{\infty} x_2 \widetilde{F_2\,(x_2)}\, dx_2 \tag{5}$$

= Mean length of the lactation period under the condition that P.P.A. succeeds lactation period.

On the other hand,

$$E(X_1) = \int_0^{x_2} x_1 \int_0^{\infty} F_1(x_1, x_2)\, dx_2\, dx_1 + \int_{x_2}^{\infty} x_1 \int_0^{\infty} F_2(x_1, x_2)\, dx_2\, dx_1 \tag{6}$$

and

$$E(X_2) = \int_0^{x_1} x_2 \int_0^{\infty} F_1(x_1, x_2)\, dx_1\, dx_2 + \int_{x_1}^{\infty} x_2 \int_0^{\infty} F_2(x_1, x_2)\, dx_1\, dx_2 \tag{7}$$

Further

$$E(X_1) = \underbrace{E(X_1 | X_2 > X_1)}_{\substack{|| \\ E(X_1)}} P(X_2 > X_1) + E(X_1 | X_2 < X_1) P(X_2 < X_1)$$

$$\Rightarrow E(X_1 | X_2 < X_1) = \frac{1}{P(X_2 < X_1)} [E(X_1) - E(X_1 | X_2 > X_1) P(X_2 > X_1)] \tag{8}$$

and

$$E(X_2) = \underbrace{E(X_2 | X_1 > X_2)}_{\substack{|| \\ E(X_2)}} P(X_1 > X_2) + E(X_2 | X_1 < X_2) P(X_1 < X_2)$$

$$\Rightarrow E(X_2 | X_1 < X_2) = \frac{1}{P(X_1 < X_2)} [E(X_2) - E(X_2 | X_1 > X_2) P(X_1 > X_2)] \tag{9}$$

$$\pi_1 = \int_0^{\infty} \int_{x_1}^{\infty} \alpha \beta' e^{-\beta' x_2 - (\alpha + \beta - \beta') x_1}\, dx_2 dx_1 = \frac{\alpha}{\alpha + \beta} \tag{10}$$

$$\pi_2 = \int_0^{\infty} \int_{x_2}^{\infty} \alpha' \beta e^{-\alpha' x_1 - (\alpha + \beta - \alpha') x_2}\, dx_1 dx_2 = \frac{\beta}{\alpha + \beta} \tag{11}$$

$$\Rightarrow \pi_1 = 1 - \pi_2 \tag{12}$$

and

$$P_1(x_1) = \int_0^{x_1} F_1(x_1, x_2)\, dx_2 + \int_{x_1}^{\infty} F_2(x_1, x_2)\, dx_2$$

$$= \frac{\alpha'\beta}{\alpha+\beta-\alpha'} e^{-\alpha' x_1} + \frac{(\alpha-\alpha')(\alpha+\beta)}{\alpha+\beta-\alpha'} e^{-(\alpha+\beta)x_1} \tag{13}$$

$$\alpha+\beta-\alpha' \neq 0$$

$$P_2(x_2) = \int_0^{x_2} F_1(x_1, x_2)\, dx_1 + \int_{x_2}^{\infty} F_2(x_1, x_2)\, dx_1$$

$$= \frac{\alpha\beta'}{\alpha+\beta-\beta'} e^{-\beta' x_2} + \frac{(\beta-\beta')(\alpha+\beta)}{\alpha+\beta-\beta'} e^{-(\alpha+\beta)x_2} \tag{14}$$

$$\alpha+\beta-\beta' \neq 0$$

$$\int_{x_1}^{\infty} P(x_2 \mid x_1)\, dx_2 = \int_{x_1}^{\infty} \frac{F_1(x_1, x_2)}{P_1(x_1)}\, dx_2$$

$$= \frac{\alpha(\alpha+\beta-\alpha')\, e^{-(\alpha+\beta)x_1}}{(\alpha-\alpha')(\alpha+\beta)\, e^{-(\alpha+\beta)x_1} + \alpha'\beta e^{-\alpha' x_1}} \tag{15}$$

$$\int_{x_2}^{\infty} P(x_1 \mid x_2)\, dx_1 = \int_{x_2}^{\infty} \frac{F_2(x_1, x_2)}{P_2(x_2)}\, dx_1$$

$$= \frac{\beta(\alpha+\beta-\beta')\, e^{-(\alpha+\beta)x_2}}{(\beta-\beta')(\alpha+\beta)\, e^{-(\alpha+\beta)x_2} + \alpha\beta' e^{-\beta' x_2}} \tag{16}$$

$$\Rightarrow \widetilde{F_1(x_1)} = \frac{\alpha'\beta e^{-(\alpha+\beta-\alpha')x_1} + (\alpha-\alpha')(\alpha+\beta)\, e^{-2(\alpha+\beta)x_1}}{(\alpha-\alpha')\, e^{-(\alpha+\beta)x_1} + \frac{\alpha'\beta}{(\alpha+\beta)} e^{-\alpha' x_1}} \tag{17}$$

$$\Rightarrow \widetilde{F_2(x_2)} = \frac{\alpha\beta' e^{-(\alpha+\beta-\beta')x_2} + (\beta-\beta')(\alpha+\beta)\, e^{-2(\alpha+\beta)x_2}}{(\beta-\beta')\, e^{-(\alpha+\beta)x_2} + \frac{\alpha\beta'}{(\alpha+\beta)} e^{-\beta' x_2}} \tag{18}$$

$$\Rightarrow \widetilde{E(X_1)} = E(X_1 \mid X_2 > X_1) = \frac{1}{(\alpha+\beta)} - \frac{(\alpha'\beta)^2}{(\alpha+\beta)\,(\alpha-\alpha'\,(2\alpha'-\alpha-\beta)^2} \tag{19}$$

= Expected P.P.A. given that lactation succeeds P.P.A.

$$\Rightarrow \widetilde{E(X_2)} = E(X_2 \mid X_1 > X_2) = \frac{1}{(\alpha+\beta)} - \frac{(\alpha\beta')^2}{(\alpha+\beta)\,(\beta-\beta')^2\,(2\beta'-\alpha-\beta)^2} \tag{20}$$

= Expected lactation given that P.P.A. succeeds lactation.

$$E(X_1) = \frac{\alpha'+\beta}{\alpha'(\alpha+\beta)} \tag{21}$$

$$E(X_2) = \frac{\alpha+\beta'}{\beta'(\alpha+\beta)} \tag{22}$$

$\Rightarrow$ $E\,(X_1 \mid X_2 < X_1)$ = Expected P.P.A. given that lactation precedes P.P.A.

$$= \frac{1}{(\alpha+\beta)}\left[\frac{\alpha'+\alpha+\beta}{\alpha'} + \frac{\alpha\beta(\alpha')^2}{(\alpha-\alpha')^2\,(2\alpha'-\alpha-\beta)^2}\right] \tag{23}$$

$\Rightarrow$ $E\,(X_2 \mid X_1 < X_2)$ = Expected lactation given that P.P.A. precedes lactation

$$= \frac{1}{(\alpha+\beta)}\left[\frac{\beta'+\alpha+\beta}{\beta} + \frac{\alpha\beta(\beta')^2}{(\beta-\beta')^2\,(2\beta'-\alpha-\beta)^2}\right] \tag{24}$$

assuming four sets of hypothetical (but plausible) values of the parameters α, β, α' and β' given in columns (1, 2, 3,) and (4) of Table 1.

We have obtained

$E\,(X_1 \mid X_2 > X_1)$, $E\,(X_1)$, $E\,(X_1 \mid X_2 < X\,1)$, $E\,(X_2 \mid X_1 > X\,2)$ $E\,(X_2)$ and $E\,(X_2 \mid X_1 < X_2\,)$

These are tabulated in Table 1.

TABLE 1

	Hypothetical Parameter Value				*Mean P.P.A. (When Lactation Period > P. P.A.)*	*Mean P.P.A. (When Lactation Period < P. P.A.) (F_1)*	*Mean P.P.A.*	*Mean Lactation Period (When P PA > Lactation Period)*	*Mean Lactation Period (When PPA < Lactation Period) (F2)*	*Mean Lactation Period*	*Prob.(of Lactation Period > P.P.A.)* π_{π_1}	*Prob. (of Lactation Period <P P A.)* π_2	F
	1	2	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
	α	β	α^I	β^I	$E(X_1 \mid X_2 > X_1)$	$E(X_1 \mid X_2 < X_1)$	$E(X_1)$	$E(X_2 \mid X_1 > X_2)$	$E(X_2 \mid X_1 < X_2)$	$E(X_2)$	π_{π_1}	π_2	F
1.	0.42	0.54	2.63	3.77	1.02	1.45	1.26	1.04	1.32	1.16	0.436	0.564	1.38
2.	0.61	0.94	3.15	3.92	0.62	0.98	0.84	0.64	0.92	0.75	0.394	0.606	0.94
3.	0.335	0.67	2.005	2.34	0.95	1.52	1.33	0.98	1.46	1.14	0.333	0.667	1.48
4.	0.75	1.33	3.495	4.65	0.46	0.77	0.66	0.47	0.72	0.56	0.361	0.639	0.74

Conclusion

Measures of infecundable period are:

F_1 = First measure: 1.45,0.98,1.52 and 0.77 (col. 6)
F_2 = Second measure = 1.32,0.92,1.46 and 0.72 (col. 9)

Estimates of mean infecundable period:

$$F = \pi_1 F_1 + \pi_2 F_2$$

The estimates of the mean infecundable following hypothetical values of the parameters of the model are given in column (13) in Table 1.

Remarks

The methodology may be tested on a live data of P.P.A. and lactation periods. In this case, we get frequency estimates of mean P.P.A. and mean lactation period, mean P.P.A. when lactation is larger and mean lactation when P.P.A. is larger etc. This may give rise to estimates of α, β, α', β' by the method of moments which can then give us estimate of the mean infecundable period is given in column (13). An observation in the diagram will show that perhaps *FI* may be slightly under-estimate to provide a measure of the infecundable period. By the same reasoning, *FT.* may be slightly overestimate. Hence $F=\pi_1 F_1 + \pi_2 F_2$ may turn out to be a reasonable measure of the true infecundable period after childbirth.

References

Biswas, S., 1988, *Stochastic Processes in Demography and Applications,* Wiley, New York.

Biswas, S. and Sehgal, V. K., 1988, A note on the efficacy of the abstinence period for preventing reconception in the context of the period of post-partum amenorrhoea, *Demography India,* XVII (1); 139-147.

Biswas, S., 1963, A study of amenorrhoea after childbirth and its relationship to lactation period, *Indian Journal of Public Health,* VII (I): 9-14.

David, H. A. and Moeschberger, M. L., 1978, *The Theory of Competing Risks,* Charles Griffin and Co. Ltd.

Freund, J. E., (1961), A bivariate extension of the exponential distribution, *J. Amr. Slat. Assoc.* 56,971-77.