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A Critical Review of Bongaarts' Aggregate Model and Development of an Index as a Suggestive Measure**

FOR ANY population at any point of time actual reproductive performance is the resultant of age pattern of fertility as well as women-age structure. It is very well known that the age pattern of fertility is the outcome of various socio-economic and cultural factors that govern the reproductive behaviour of the population. On the other hand, age distribution of population is the resultant of past fertility and mortality.

Bongaarts (1983), in his Aggregate Model, has shown that nearly all variations in the levels of fertility come from the variations in the four major proximate determinants, namely— proportion-married, contraception, induced abortion and post-partum infecundability. The fertility effects of these four are measured in the model by four indices—C_m, Cc, C_a and C_i respectively as:

$$TFR = C_m * C_c * C_a * C_i * TF; \qquad \text{(i)}$$

where *TFis* the Total Fecundity.

All the proximate determinants mentioned above affect age-pattern of fertility. Some of them are affected by the age pattern of fertility also. In such situation, many time there is likelihood of over or under estimating the contribution of such determinants toward fertility changes. Moreover, as *TFR is* age-standardised, it does not reflect age structure and hence it is totally insensitive to changes in age structure. So in the model variations in 77-7? is explained by variations in age-pattern of fertility only.

However, while decomposing *CBR,* Bongaarts (1983) has given an index, which (according to him) takes into account age-sex composition. In his formulation *CBR is* related to the proximate determinants as:

$$CBR = S * C_m * C_c * C_a * C_i * TF; \qquad \text{(ii)}$$

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where S is an age-sex composition factor calculated as:

$$S = \frac{CBR}{TFR} \qquad \text{(iii)}$$

He mentioned that variations in S are caused by changes in population's age-sex structure. But variations in S not only come from changes in age-structure but also from changes in age-pattern of fertility. So measuring the impact of age-structure on fertility by this formula will be either under or over estimated.

Shortcomings Associated with '*S*'

If each *ASFR* in a schedule is divided by the summation of *ASFR*s, it gives a distribution, which reflects the age pattern of fertility. Such a distribution may be termed as *ADN*. Similarly age distribution of women within the reproductive span can be computed. Such a distribution may be termed as *WDN*.

Now, if A_is are *ASFR*s, then *ADN* is given as

$$a_i = \frac{A_i}{\sum A_i};$$

and if W_is are number of women in each age-group in the reproductive period, then *WDN* is given as

$$w_i = \frac{W_i}{\sum W_i};$$

$i = 1, 2, \ldots, 7.$

Now, we can define one variable:

$$Q = \sum_{i=1}^{7} a_i * w_i; \qquad \text{(iv)}$$

which incorporates both the age pattern of fertility and women-age distribution.

Again, by definition $A_i = B_i / W_i$, where B_is are number of births in each age group in the reproductive period. So,

$$a_i * w_i = \frac{B_i / W_i}{\sum A_i} * \frac{W_i}{\sum W_i}$$

$$= \frac{1}{\sum A_i} * \frac{B_i}{\sum W_i}$$

And,

$$\sum a_i * w_i = \frac{1}{\sum A_i} * \frac{\sum B_i}{\sum W_i}.$$

By definition,

$$\sum B_i / \sum W_i = GFR; \text{ and } \sum A_i = \sum ASFR.$$

So,

$$Q = \frac{GFR}{\sum ASFR}; \qquad \text{(v)}$$

which is very similar to S (= CBR/TFR) as proposed by Bongaarts.

Now, GFR can be expressed in terms of CBR as follows:

$$GFR = \frac{\text{Total Live Births}}{\text{Women (15-49)}}$$

$$= \frac{CBR * \text{Total Population}}{\text{Total Population} * \text{Proportion Women (15-49)}}$$

$$= \frac{CBR}{\text{Proportion Women (15-49)}}$$

Or,

$$CBR = \text{Proportion Women (15-49)} * GFR$$

Now,

$$S = \frac{CBR}{TFR}$$

$$= \frac{\text{Proportion Women (15-49)} * GFR}{5 * \sum ASFR}$$

If Proportion – Women (15–49) = k, then

$$S = m * Q; \qquad \text{(vi)}$$

where $m = k/5$.

Mathematically, it is shown that S is very similar to Q [as in equation—(vi)] and as Q measures changes in both the age pattern of fertility as well as women-age structure [as in equation—(iv)] S is also supposed to do that. So variations in S will not show true variations in age structure.

In view of such measurement problems, there is need to have an index that constitutes the effect of both i.e. age structure of population and age pattern of fertility. One may develop separate index of these factors and use the concept of multiplicative modelling to have one single index that describes the changes in fertility over time.

The New Proposed Index

Most of the governments in developing countries have objectives to achieve a stable population with replacement level of fertility and maintain it for a considerable time. The index, which may be given a name as I, will reflect fertility transition taking into account changes in ADN and WDN with respect to any reference population(s) which is stable with replacement level of fertility. So development of I requires selection of a reference or desired ADN and a reference WDN. While selecting those one should consider a country's past experience as well as (projected) future trends. The State of Kerala achieved $TFR = 2.0$ in 1989. So, the age pattern of fertility of Kerala in 1989 can be taken as reference pattern (ADN). Adverting to WDN, it can be seen that Tamil Nadu - Population in 2016 (Report of the Technical Group on Population Projection, 1996) has an almost equal age distribution in the reproductive period. So that distribution can be taken as reference WDN. At any point of time I will measure Kullback distance between the study and reference ADNs [say, $K(A)$]

and the same between study and reference *WDN*s [say, $K(W)$]. I will be multiplication of the above mentioned quantities.

Kullback distance has its root in Information Theory where it measures distance between two distributions forming Marcov Chain. According to Schoen and Kim (1991), if $q(x, t)$ is the probability distribution for the observed population at time t, and $s(x)$ is the limiting probability distribution for the stable population, the Kullback distance between the q and s distributions, $K(t)$ is defined as

$$K(t) = \int_0^{\infty} q(x,t) * \ln[q(x,t)/s(x)]dx; \tag{vii}$$

where 'ln' indicates natural logarithm.

Application of Kullback distance technique in the present study requires the assumption that reference distributions are nothing but the ultimate phases of present distributions.

Using the notations used earlier, the Kullback distance between the discreet study and reference *ADN*s can be calculated as (taking absolute values only):

$$K(A) = \sum_{i=1}^{7}(a_i) * [\ln(a_i / a_i^*)]; \tag{viii}$$

where a_is with asterisk belong to reference *ADN*. Similarly for the discreet *WDN*s Kullback distance can be calculated as (taking absolute values only):

$$K(W) = \sum_{i=1}^{7}(w_i) * [\ln(w_i / w_i^*)]; \tag{ix}$$

where w_is with asterisk belong to reference *WDN*.

The index (I) is then,

$$I = K(A) * K(W) \tag{x}$$

Structure of Change in I According as $K(A)$ and $W(A)$

In this section we will convert the multiplicative model into an additive model to measure relative contribution of $K(A)$ and $K(W)$ to the changes in I separately.

For any two time points (say, 1 and 2) we have

$$I_1 = K_1(A) * K_1(W),$$
$$I_2 = K_2(A) * K_2(W), \text{ and}$$
$$I_2/I_1 = [K_2(A)/K_1(A)] * [K_2(W)/K_1(W)] \tag{xi}$$

Now, if

$(I_2/I_1) - 1$ = proportional change in I, P_I;

$[K_2(A)/K_1(A)] - 1$ = proportional change in $k(A)$, $P_{k(A)}$; and

$[K_2(W)/K_1(W)] - 1$ = proportional change in $K(W)$' $P_{k(W)}$, then

equation (xi) can be written as

$$P_I = P_{k(A)} + P_{K(W)} + R, \tag{xii}$$

where $R = P_{K(A)} * P_{K(W)}$, an interaction factor.

Numerical Applications and Discussions

Numerical application is based on a data set of fifteen major States of India for four time points: 1980, 1985, 1990 and 1994 (or mentioned otherwise). Data used in this section are:

Age Specific Fertility Rates *(ASFRs),* Women-Age Distribution within the reproductive period, *GFR* and *TFR*. Sources of data are Sample Registration System (SRS) for the above mentioned time points (or mentioned otherwise) and Report of The Technical Group on Population Projection, August 1996.

Table 1 shows a Q value of 0.159945 *for AND and WDN of Andhra* Pradesh in 1980. If *WDNis* held constant and another *ADN* is assumed, say, ACWofAndhra Pradesh in 1994 then *Q* value changes to 0.171393 (a 7.16 per cent change). It proves that *Q* is sensitive to age pattern of fertility. And, as *S* and *Q* are directly related (as in equation-vi), *S* is sensitive to age pattern of fertility also. Moreover, it is to be mentioned that as a by-product of this study, equation (vi) provides one measure:

k=CBR/GFR;i.e.,

for any population *ifCBR and GFR* are known for a particular time, proportion of women in the reproductive period can be known.

TABLE 1: *Q* VALUES FOR ANDHRA PRADESH FOR DIFFERENT *ADNs* BUT SAME *WIW*

Age-Group	*ADN-80*	*ADN-94*	*WDN-80*	*ADN-80* WDN-80*	*ADN-94* WDN-80*
15-19	0.15697	0.20573	0.22891	0.035932	0.047093
20-24	0.29991	0.41618	0.17737	0.053195	0.073817
25-29	0.25142	0.21988	0.14012	0.035228	0.030809
30-34	0.14783	0.09881	0.13046	0.019285	0.012890
35-39	0.0926	0.04054	0.11637	0.010775	0.004717
40-44	0.03391	0.01358	0.11717	0.003973	0.001591
45-49	0.01735	0.00528	0.08959	0.001554	0.000473
Total	1	1	1	0.159945	0.171393

Though both *S* and *Q* are easy to compute, one cannot decompose individual impact of *ADN or WDN* from them. However, for a hypothetical case, if a population has exactly equal number of women in each age group in the reproductive period then *Q* will show a value exactly equal to 0.142857. From this information one can guess the shape of age distribution.

The new index, *I* has been derived as in equation (x). Now, as *K(A)* and *K (W)* measure distance between study and reference distributions, both *K (A)* and *K* (W)-values will be 0 if study and reference distributions are identical. It means that 0 is the minimum desired value for *K (A) and K* (*W*) and hence *I* also.

Table 2A and 2B show *K. (A), K{W)* and /-values for fifteen major States of India for 1980, 1985, 1990 and 1994. Displayed values in all the three columns show respective distance from the desired state and distance varies according to magnitude of values. For example, *K(A)-va[ue* for Andhra Pradesh (AP) in 1980 is 0.40899, which shows a positive distance or difference from the desired level, 0. In other words there is a difference between the observed and reference age pattern of fertility for AP and the measure of that distance or difference is 0.40899.

Now, the value *of K(A)* is lowest for Kerala (KE) for all the four time points and highest for UP in 1980 and 1994, and Bihar (BI) in 1985 and 1990. It means age pattern of fertility is

Closer to the reference pattern in Kerala than those of UP and Bihar. Among other States Assam, Rajasthan and MP also have quite high values.

TABLE 2A: *K (A)*. *K(W)* AND INDEX (*I*) FOR FIFTEEN MAJOR STATES OF INDIA IN 1980, 1985

	1980			*1985*		
State	*K(A)*	*K(W)*	*Index*	*K(A)*	*K(W)*	*Index*
AP	0.40899	0.32416	0.13258	0.38092	0.26744	0.10187
AS	0.47494	0.44297	0.21038	0.46268	0.42940	0.19867
BI	-	-	-	0.59509	0.22901	0.13628
GU	0.26578	0.35021	0.09308	0.16720	0.31843	0.05324
HA	0.37025	0.41591	0.15399	0.24674	0.37553	0.09266
KA	0.41362	0.35892	0.14846	0.29862	0.30728	0.09176
KE	0.21789	0.35965	0.07837	0.07648	0.37420	0.02862
MP	0.41730	0.33172	0.13843	0.42318	0.27576	0.11670
MA	0.28105	0.31410	0.08828	0.18811	0.27159	0.05109
OR	0.37267	0.34848	0.12987	0.26766	0.31543	0.08443
PU	0.40814	0.34131	0.13930	0.23369	0.33144	0.07745
RA	0.53995	0.33129	0.17888	0.46367	0.29537	0.13696
TN	0.30633	0.27017	0.08276	0.15787	0.26889	0.04425
UP	0.62987	0.32607	0.20538	0.53166	0.24661	0.13111
WB	-	-	-	0.35764	0.33341	0.11924

TABLE 2B: *K(A)*. *K(W)* AND INDEX (7) FOR FIFTEEN MAJOR STATES OF INDIA IN 1990,1994

	1990			*1994*		
State	*K(A)*	*K(W)*	*Index*	*K(A)*	*K(W)*	*Index*
AP	0.36466	0.27223	0.09927	0.36633	0.27930	0.10232
AS	0.39339	0.41068	0.16156	0.37789	0.34608	0.13078
BI	0.54950	0.25174	0.13832	0.48138	0.19838	0.09549
GU	0.10084	0.30792	0.03105	0.07220	0.29182	0.02107
HA	0.17665	0.38277	0.06761	0.08502	0.31704	0.02695
KA	0.19587	0.31988	0.06265	0.07326	0.27058	0.01982
KE	0.05446	0.32541	0.01772	0.05858	0.26491	0.01552
MP	0.36152	0.30576	0.11054	0.32188	0.29302	0.09432
MA	0.20779	0.27011	0.05612	0.11659	0.26798	0.03125
OR	0.27747	0.33512	0.09299	0.19676	0.31474	0.06193
PU	0.10936	0.30148	0.03297	0.07454	0.27963	0.02084
RA	0.39949	0.31604	0.12624	0.37576	0.27065	0.10170
TN	0.13252	0.26631	0.03529	0.08244	0.26089	0.02151
UP	0.54820	0.29154	0.15982	0.52427	0.28528	0.14956
WB	0.35209	0.29259	0.10302	0.22404	0.25993	0.05823

Among *K* (W)-values, Tamil Nadu has the lowest in 1980 and Bihar in 1985, 1990 and 1994 (1980-data for Bihar are not available); and Assam has the highest for all the four time points. Now, these imply that women age distributions within the reproductive period in TN and BI are closer to the reference pattern and hence are more equal. However, the same in

Assam is more unequal (highly skewed). Though *K (A)-values* are minimum for Kerala, *K (W)*- values are quite high. On the other hand though Bihar has very high *K (A)-va\ues, K(W)*- values are minimum.

Now, both the *K(A)* and *K(W)* together determine /. Though *K{W)*- values for Kerala are quite high, thanks to the lowest *K(A)-values* /assumes minimum values. Among other States Assam has the highest in 1980,1985,1990 and UP in 1994. As 0 for/represents the desired state with replacement level of fertility, which most of the governments in developing countries want to achieve and maintain for a long time, the lowest /-values for Kerala imply that it is closer to that state and States like Assam and UP with higher (or highest) /-values are far away from that state.

Trends in *K (A\ K (W)* and / are clear from Table 2A and 2B. Both *K (A)* and / show a sharp (exponential) decline for Kamataka, Kerala, Punjab and Tamil Nadu. Gujarat and Haryana also show a sharp decline. Figure 9 shows trends in few selected States from 1980 to 1994. On the other hand *K. (W)* shows a very slow decline for most of the States. Proportional changes in /due to decline in *K(A)* and *K(W}* (also due to increase in interaction factor) from 1980 to 1994 are shown in Table 3.

TABLE 3: PROPORTIONAL CHANGE IN/ACCORDING AS *K (A}* AND *K (W)* FOR FIFTEEN MAJOR STATES OF INDIA (1980-1994)

State	(-)$P_{K(A)}$	(-)$P_{K(W)}$	(+)$P_{K(A)}$ *$P_{K(W)}$	(-)P_1
AP	0.10431	0.13839	0.01443	0.22826
AS	0.20434	0.21873	0.04470	0.37837
BI*	0.19108	0.13375	0.02556	0.29927
GU	0.72835	0.16673	0.12144	0.77364
HA	0.77037	0.23772	0.18313	0.82496
KA	0.82288	0.24613	0.20253	0.86647
KE	0.73115	0.26342	0.19260	0.80197
MP	0.22866	0.11666	0.02668	0.31865
MA	0.58516	0.14683	0.08592	0.64607
OR	0.47203	0.09682	0.04570	0.52314
PL)	0.81737	0.18072	0.14771	0.85037
RA	0.30408	0.18304	0.05566	0.43147
TN	0.73088	0.03435	0.02510	0.74012
UP	0.16765	0.12510	0.02097	0.27178
WB*	0.37356	0.22039	0.08233	0.51162

*1985-1994

As *K(A)* measures changes in age pattern of fertility with respect to the reference pattern, and as *TFR* is sensitive to age pattern of fertility, variations in *TFR* can be explained by variations in *K(A).* Similarly, variations in *GFR* can be explained by variations in *I* or in *K(A)* and *K(W).* Cross sectional consistencies have been checked graphically (Fig. 1 to Fig. 8). Regression analyses have also been done to show (in Table 4) the amount of variations in *TFR* [and *GFR]* explained by variations in *K (A)* [and *K(A), K(W}}.* However, these analyses have limitations. As number of observations are less (4 only) with respect to independent variable(s), the differences between R^2 and adjusted R^2, and levels of significance are quite high.

TABLE4: SPSS OUTPUT OF REGRESSION ANALYSES: (1980-1994)

State	*TFR: K (A)*			*GFR: K (A), K (W)*		
	R^2	*Adj.* R^2	*Sig. F*	R^2	*Adj.*R^2	*Sig. F*
AP	0.66834	0.50251	0.1875	0.91189	0.73568	0.2968
AS	0.55583	0.33375	0.2545	0.84518	0.53555	0.3935
BI	0.73369*	0.60054	0.1434	0.68406+	0.05217	0.5621
GU	0.99843	0.99765	0.0008	0.96093	0.88278	0.1977
HA	0.64659	0.46989	0.1959	0.96166	0.88497	0.1958
KA	0.80731	0.71097	0.1015	0.64369	-	0.5969
KE	0.82998	0.74497	0.0890	0.99807	0.99420	0.0440
MP	0.47609	0.21413	0.3100	0.90944	0.72832	0.3009
MA	0.78201	0.67302	0.1157	0.71071	0.13212	0.5379
OR	0.83888	0.75832	0.0841	0.98870	0.96611	0.1063
PU	0.97078	0.95618	0.0147	0.94480	0.83439	0.2350
RA	0.85145	0.77718	0.0773	0.73163	0.19488	0.5180
TN	0.93112	0.89669	0.0351	0.97232	0.91695	0.1664
UP	0.62548	0.43822	0.2091	0.95121	0.85363	0.2209
WB	0.94905 *	0.92358	0.0258	0.97568"1"	0.92703	0.1560

*1985-1994;+1981-1994

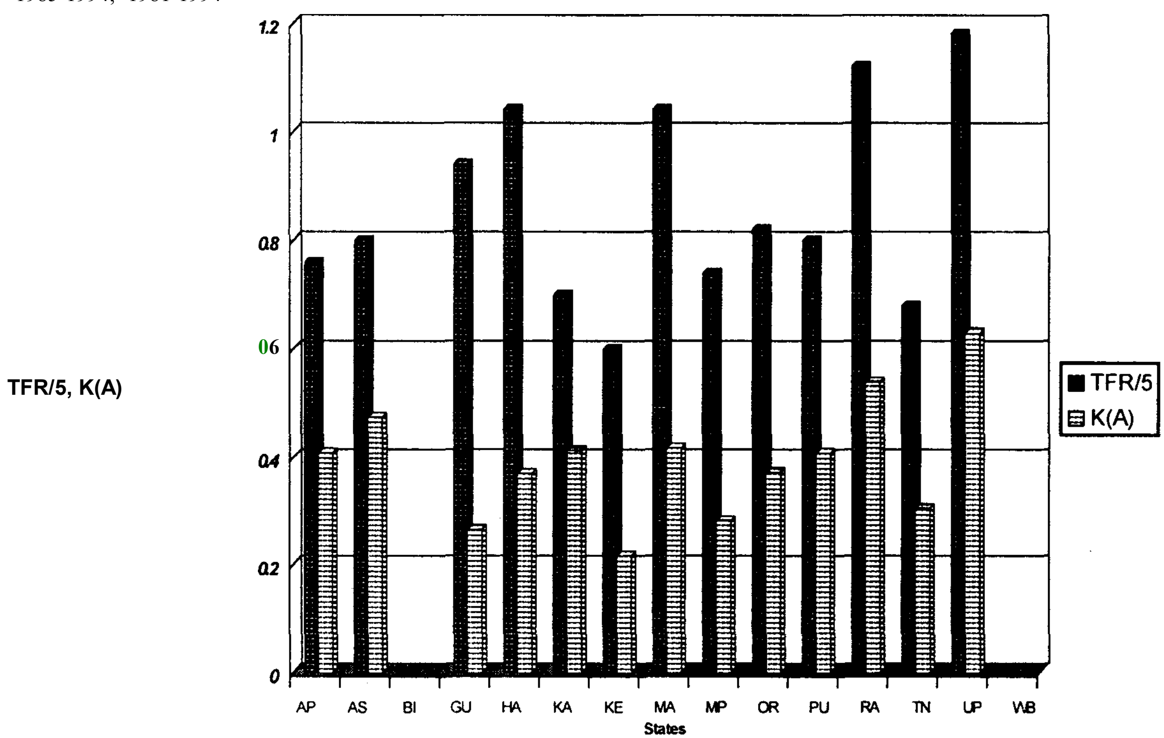

Fig. I. TFR and K(A) in Fifteen Major States of India, 1980

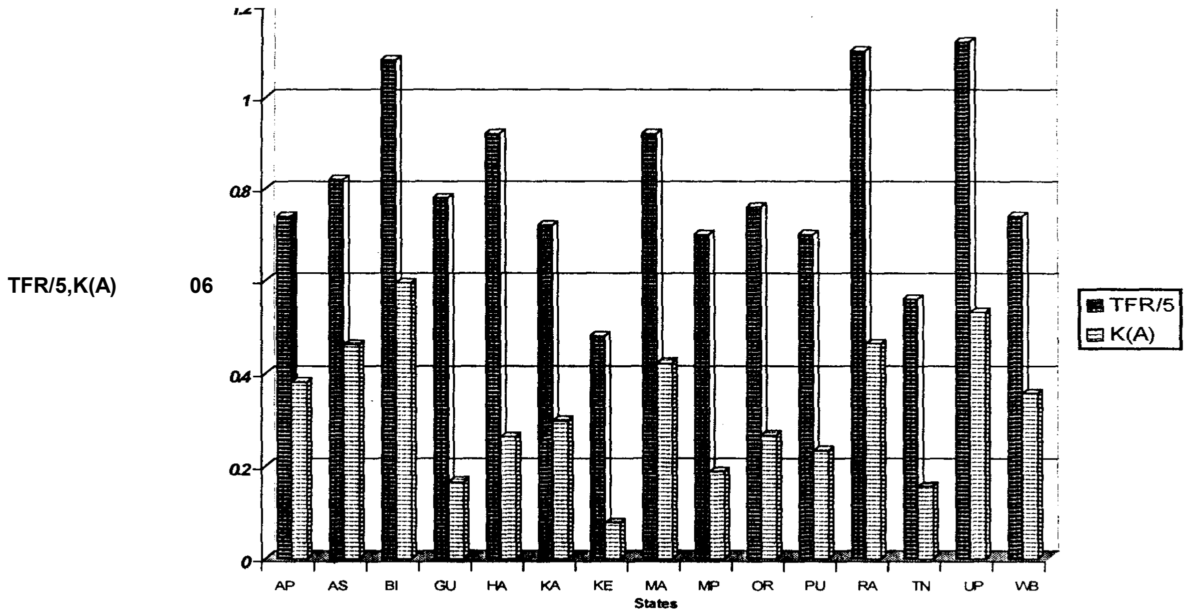

Fig. 2. TFR and K(A) in Fifteen Major States of India, 1985

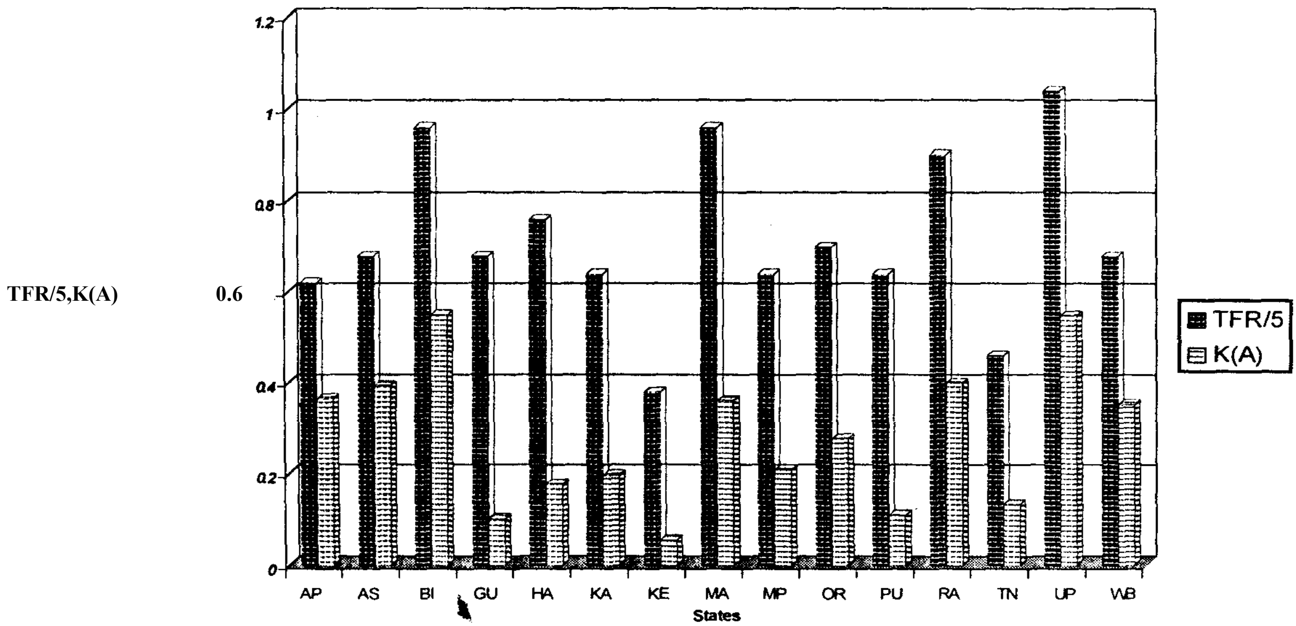

Fig. 3. TFR and K(A) in Fifteen Major States of India, 1990

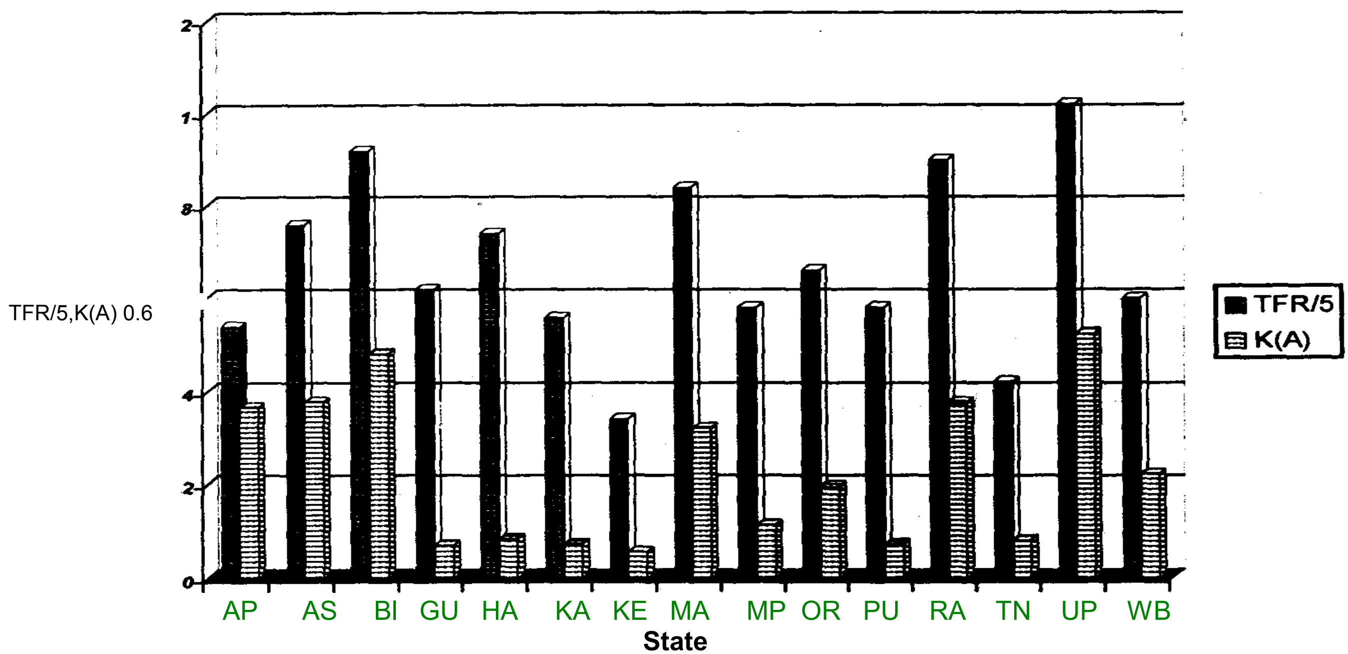

Fig. 4. TFR and K(A) in Fifteen Major States of India, 1994

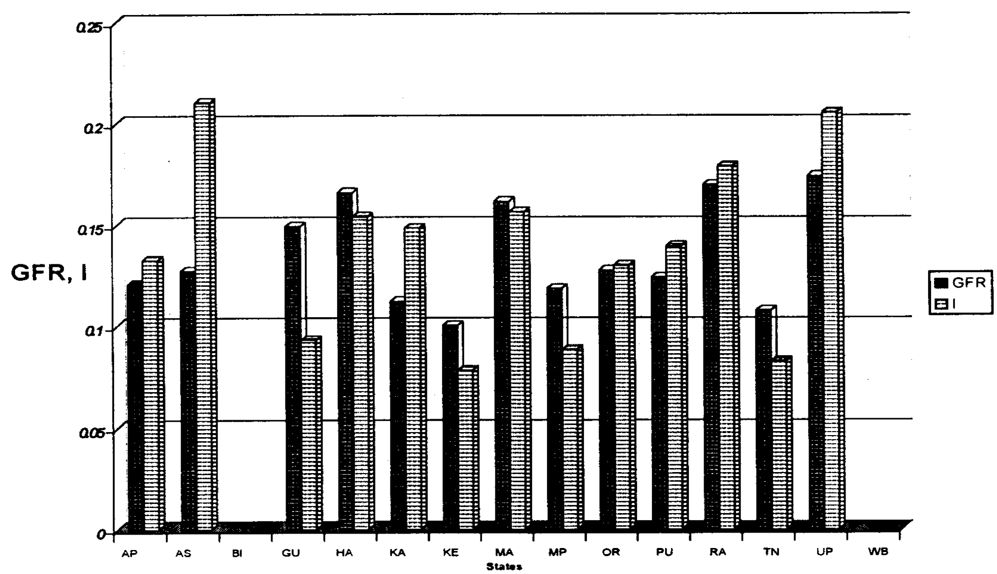

Fig. 5. GFR and I for Fifteen Major States of India, 1980

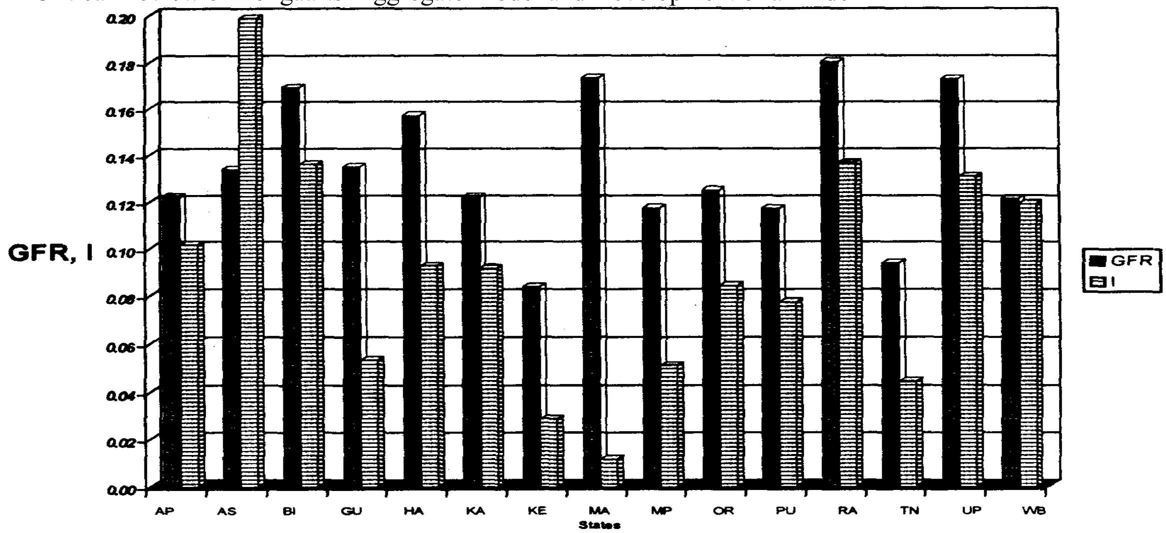

Fig. 6. GFR and I for Fifteen Major States of India, 1985

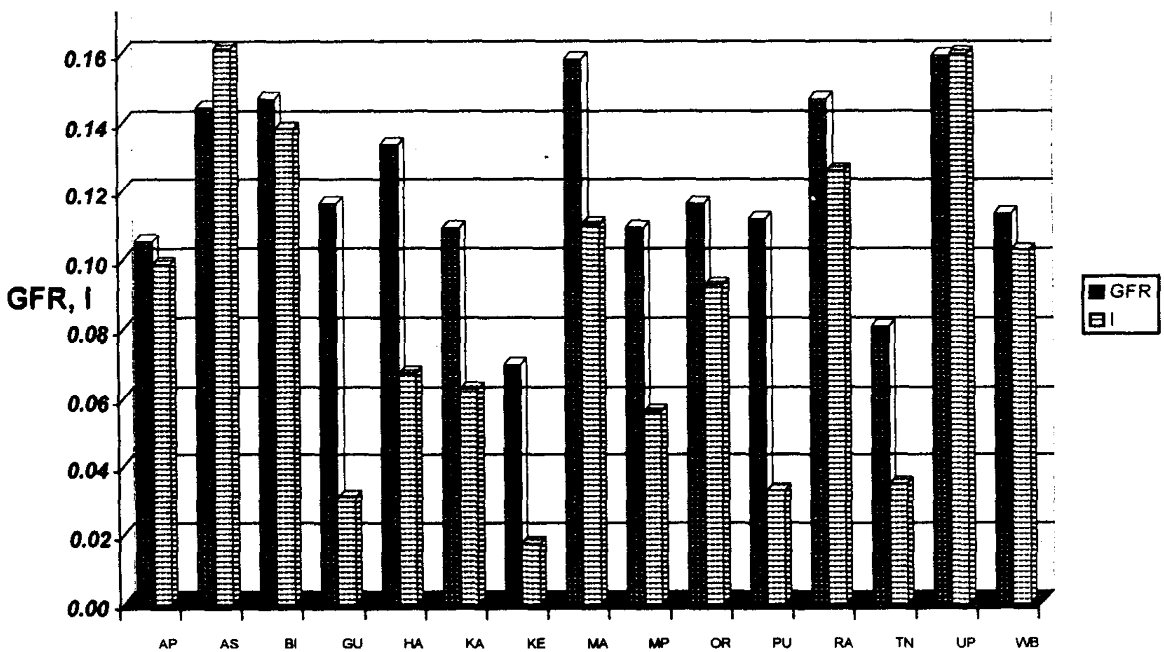

Fig. 7. GFR and I for Fifteen Major States of India, 1990

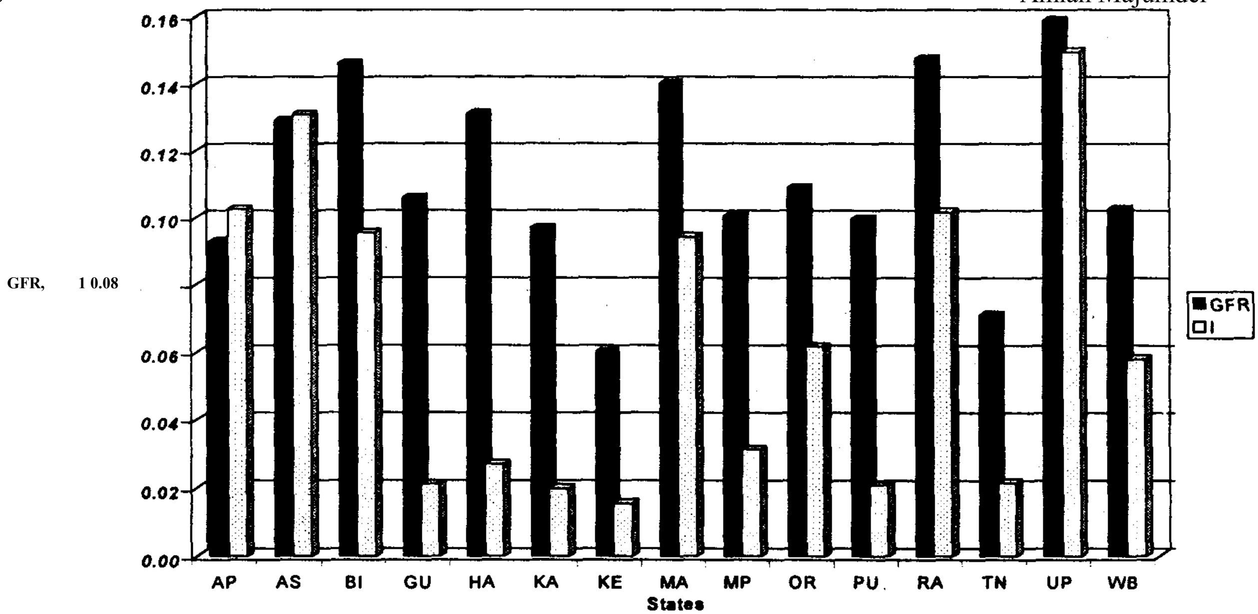

Fig. 8. GFR and I for Fifteen Major States of India, 1994

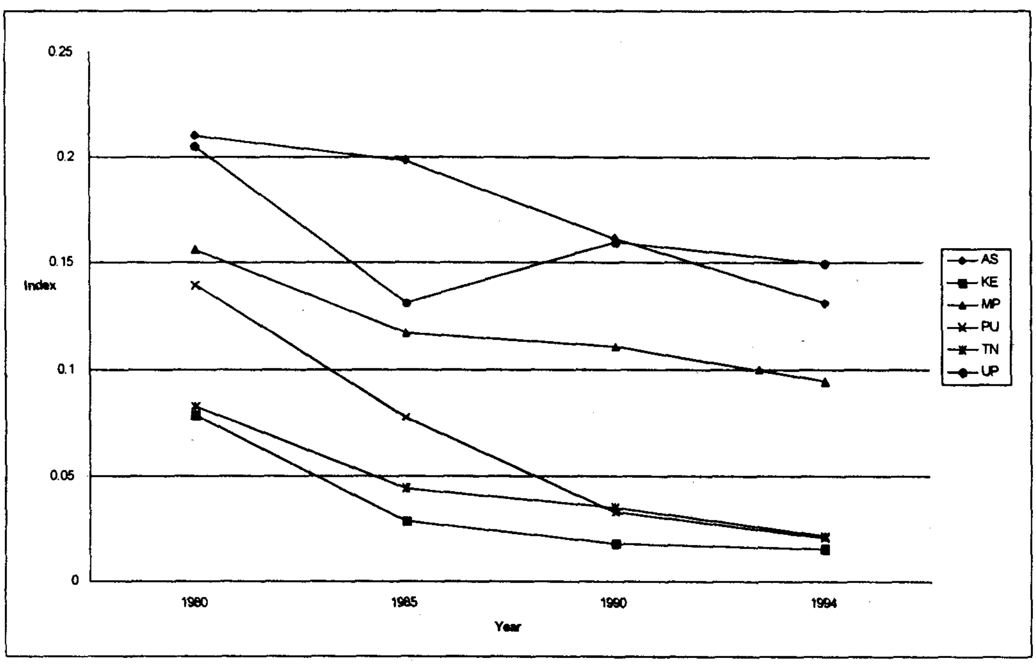

Fig. 9. Trends in I for Few Selected States in India, 1980 1985 1990 1994

Conclusion

While exploring all the possibilities to locate variations in age structure and age-pattern of fertility with fertility changes over time (and differentials at a point of time), this study find comfortable to use the Kullback distance technique. Kullback distance technique has been used in demography for the first time by Tuljapurkar (1982). Although no demographic interpretation of Kullback distance has been provided, it is a meaningful demographic quantity, which measures distance between two distributions (Schoen and Kirn, 1991). The index proposed here can be used in studies of fertility transition and fertility differentials.

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