Demography India Vol. 26, No. 2 (1997), pp.
287-296

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Refined Estimates of Person Years Lived at Infant and Childhood Ages of Model Life Tables

Introduction

TN developing countries, data on mortality are invariably incomplete and loaded with errors. Under such circumstances model life tables are used to identify the level of mortality. These model life tables, which help estimating the level and the age pattern of mortality for countries where only incomplete data are available, are constructed based on the mortality experiences of several populations for which life tables are available. These models have been constructed after careful evaluation and analysis of available data.

However, errors of approximation in computing several indices used in model life tables are likely to exist. Further at the user level, errors in identifying the model and the level of mortality may occur. These errors, though occasionally may cancel each other out, generally add up and the estimate is likely to deviate substantially from true values and hence in this paper an attempt has been made to reduce one of these errors - the error of approximation.

In this paper improved approximations for the person years lived at infant and childhood ages are obtained for the most commonly used model life tables, namely Regional Model Life Tables (Coale and Demeny, 1983) and Model Life Tables

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For Developing Countries (United Nations, 1982). Using Weibull parameterisation, the refined estimates are arrived at and compared with original.

Choice of Model

The number of person years lived by a cohort of $/_0$ persons in the age interval $(x, x + n)$ say ${}_nL_x$ depends upon the functional form of $/_x$ between x and $x + n$. In case of $n = 1$ for ages above 5 years, linearity assumption is fairly valid as the shape of $/_x$ is close to linearity and the interval is also small. The linear approximation is certainly inappropriate for ages below 5 years even for $n = 1$ where the change in hazard rate is sharp and $/_x$ curves sharply. At the same time efforts to identify a single mathematical expression which is capable of describing the entire life span were not successful. It is, therefore, necessary to identify the functional approximation for $/_x$ to specific intervals of age to obtain a fairly accurate ${}_nL_x$. In this paper we are concerned with the age interval of 0-5 years.

Keyfitz (1966) proposed a hyperbolic model for the survival probability, $/_x$, at early ages of life. Later Hartman (1980) proposed a logarithmic approximation model for $/_x$. The Weibull hazard function which has been widely used in reliability studies for a long time has been recommended by Choe (1981) for modelling infant and child motrality experiences. Choe has also shown Weibull function's superiority over the Gompertz model. Comparing these models in their ability to fit infant and child mortality experience, Krishnamoorthy (1982) concluded that Weibull provides the best fit. For this purpose he used data from Australia, a developed country. Pathak (1991) found that the Weibull function fits well with the survival pattern in the infant and childhood ages in Indian situation. Mathew (1995) on the basis of very reliable data generated from a large longitudinal community survey conducted at South India, found that Weibull provides a very good fit for graduating infant and childhood mortality even in developing countries. Hence it is concluded that the monotonically decreasing mortality at infant and childhood ages is well described by the Weibull function for both developed and developing countries.

Model Life Tables

The most commonly used two sets of model life tables at present are Regional Model Life Table by Coale and Demeny (1983) and Model Life Tables for Developing Countries by United Nations (1982). In order to meet the need for describing varying age pattern of mortality, Coale and Demeny published four sets of model life tables. ${}_1L_0$ was calculated using $K_0\, /_0+(1- K_0\,)\, /_1$, and ${}_4L_1$ was calculated using

$K_1\, l_1 + (4 - K_1)\, l_5$, where the separation factors K_0 and K_1 were related to the average age at death of those who die in the age interval and are empirically determined for each sex, region and mortality level combinations (Coale and Demeny, 1983:
22-23)

The weights (K_0 and K_1 the "separation factors") that relate ${}_1L_0$ to l_0 and l_1 and ${}_4L_1$ to l_1 and l_5 can be shown to equal the age at death of those members of the life table population who die under the age 1 (for K_0), and the age at death minus 1 of those who die between ages 1 to 4 (for K_1). The values of the factor K_0 were, therefore determined by examining the average age at death under 1 year in the records of the populations whose mortality experience was the basis for the four regional model life tables.

The value of K_1 in the expression ${}_4L_1 = K_1 l_1 + (4 - K_1) l_5$ was determined from the estimates of l_2 l_3 and l_4. ${}_4L_1$, was calculated on the assumption that l_x can be considered linear in each single-year age interval from 1 to 5; thus $K_1 = 0.5 + \alpha_2 + \alpha_3, + \alpha_4$ when $l_2 = \alpha_2 l_1 + (1 - \alpha_2)\, l_5$, etc. Each α_i. was based on the relation of l_i to l_1 and l_5, observed in the life tables upon which each family of model table was based.

In the Model Life Tables for Developing Countries (U.N., 1982), ${}_nL_x$ was calculated using ${}_nL_x = {}_na_x \,.\, l_x + (n - {}_na_x)\, l_{x+n}$ where ${}_na_x$ is the average number of years lived in the age interval $(x, x + n)$ by those dying during the age interval. For ages under 5, ${}_na_x$ values were taken from the Coale and Demeny West Region relationships. That is when the probability of dying in the first year of life ${}_1q_0 \geq 0.100$, ${}_1a_0 = 0.33$ for males and 0.35 for females and ${}_4a_1 = 1.352$ for males and 1.361 for females. When ${}_1q_0 < 0.100$, ${}_1a_0 = 0.0425 + 2.875\ {}_1q_0$ for males and ${}_4a_1 = 1.524 - 1.625\ ,{}_1q_0$ for females.

Description of Method Used

Weibull hazard function which is found to fit for explaining the entire variations in the early ages of life is used to obtain a better approximation for ${}_1L_0, {}_1L_1, {}_1L_2, {}_1L_3, {}_1L_4$ and ${}_4L_1$ of model life tables. Weibull hazard function is given by

$$\mu(X) = \beta\, \alpha\, x^{\alpha-1}, \text{ when } \chi \geq 0, \text{ and} \qquad 1$$

$$= 0, \text{ Otherwise.}$$

The survival function is given by

$$l_x = e^{-\beta x \alpha} \qquad 2$$

The fit of this Weibull is examined in the Appendix (Tables A-l and A-2) in which α and β are estimated from the single year I_x values from 0 to 5. For the reasons given in the Appendix, the same approach is not followed here. Instead it is proposed to estimate these parameters directly from / and /, only.

Given the values of ^ and *l*^

$$l_1 = e^{-\beta} \quad \textbf{and}$$

$$l_5 = e^{-\beta 5^{\alpha}}$$

the values β of and α are computed as

$\beta = - l_n l_1$ and

$$\alpha = \frac{\ln\left(\dfrac{\ln l_5}{\ln l_1}\right)}{\ln 5}$$

Where β is the level parameter determining the level of infant and childhood mortality and a is the shape parameter determining the relative levels of infant and childhood mortality.

As ${}_nL_x = {}_x\int^{n+x} l_t dt$, where $l_x = e^{-\beta\chi^{\alpha}}$

is not integrable for all α, we have to resort to numerical solutions for α and β estimated from l_1, and l_5, using (3).

As our interest is to provide better approximation for ${}_1L_0$, ${}_1L_1$,, ${}_1L_2$ ${}_1L_3$, ${}_1L_4$, and ${}_4L_1$ we tried numerical integration by splitting the age intervals into equal bits of 10, 20, 50, 100, 200 and 250 and the solutions are obtained assuming step function within these small intervals. Trials showed that, for obtaining close to accurate solution, the interval *(x, x + n)* need be split about 200 bits. The computation is carried out to the selected two sets of model life tables.

Findings

For selected levels we have compared the differences between originally published and computed values. The findings are displayed in Tables 1 and 2. From

TABLE 1: DIFFERENCES BETWEEN ORIGINALLY PUBLISHED BY COALE AND DEMENY AND THE IMPROVED ESTIMATES* FOR FEW SELECTED LEVELS

Region and Sex.	*Levels*	e^0_0	${}_1L_0$	${}_1L_0$ *	*Diff.*	${}_4L_1$*	${}_4L_1$*	*Diff.*
West	3	25.0	80139	76009	4130	238224	241360	- 3136
Female	8	37.5	87285	84768	2517	293814	296347	- 2533
	13	50.0	92279	90684	1595	335667	337261	- 1594
	18	62.5	95350	95016	334	369479	369885	- 406
	23	75.0	98617	98598	19	393280	393287	7
West	3	28.8	76433	7)400	5033	222444	225188	- 2744
Male	8	34.8	84753	81703	3050	282022	284340	- 2318
	13	47.0	90609	88632	1977	327678	329135	- 1457
	18	58.8	94445	93700	745	363390	363776	- 386
North	3	25.0	82621	80273	2348	249237	249792	- 555
Female	13	50.0	93108	92145	963	338943	339524	- 581
	18	62.5	95544	95549	- 5	369316	369271	45
	23	75.0	98322	98330	- 8	391536	391473	63
North	3	22.3	79175	76337	2838	232538	232872	- 334
Male	8	34.4	86547	84851	1696	288629	289330	- 701
	13	46.6	91645	90522	1123	330847	331392	- 545
	18	58.8	94759	94532	227	363996	363859	137
	23	71.5	97851	97854	- 3	389106	388977	129
East	3	25.0	75013	70094	4919	220353	223202	- 2849
Female	8	37.5	83642	80631	3011	279474	281848	- 2374
	13	50.0	89829	87796	2033	325933	327368	- 1435
	18	62.5	94165	93393	772	363301	363759	- 458
	23	75.0	98019	98003	16	390791	390809	18
East	3	22.4	69620	63385	6235	197191	199648	- 2457
Male	8	34.7	80043	76315	3728	262936	265128	- 2192
	13	46.7	87485	85026	2459	314466	315858	- 1392
	18	58.3	93189	91805	1384	356289	356809	- 520
	23	70.2	97470	97419	51	388215	388211	4
South	3	25.0	82819	81507	1312	232801	241216	- 8415
Female	8	37.5	87966	86758	1208	284274	290352	- 6078
	13	50.0	91487	90146	1341	323615	327023	- 3408
	18	62.0	94024	92901	1123	355531	356714	- 1183
	23	75.0	96587	96264	323	381445	381600	- 155
South	3	24.6	80608	78964	1644	227674	234974	- 7300
Male	8	36.2	86341	84961	1380	278922	284393	- 5471
	13	47.3	90268	88838	1430	318140	321386	- 3246
	18	58.5	93419	91998	1421	350633	351926	- 1293
	23	70.9	96165	95760	405	379324	379428	- 104

Diff. - Difference

in both Coale Demeny Regional Model Life Tables and in U.N. Model Life Tables. these tables it is clear that ,Lp is generally over estimated and L is under estimated In both model life tables, the over estimation of^Lg increases and the under estimation of^L, increases with increasing mortality. As regards Regional Model Life Tables, the over estimation of ,L(, is highest in the East Model, particularly for males. On the other hand, the under estimation of^L, is highest in the South Model, particularly for females.

TABLE 2: DIFFERENCES BETWEEN ORIGINALLY PUBLISHED AND THE IMPROVED ESTIMATES* FOR FEW SELECTED LEVELS OF U.N. MODEL LIFE TABLES (GENERAL PATTERN)

Sex.	*Levels*	e^0_0	$_1L_0$	$_1L_0$ *	*Diff.*	$_4L_1$*	$_4L_1$*	*Diff.*
Male	5	39.0	88202	86251	1951	304485	306859	- 2374
	10	44.0	90065	88240	1825	321115	322947	- 1832
	15	49.0	91796	90124	1672	336262	337616	- 1354
	20	54.0	93363	91919	1444	350031	350960	- 929
	25	59.0	94383	93628	755	362584	363004	- 420
	30	64.0	95569	95240	329	373540	373678	- 138
Female	5	39.0	90374	89187	1187	311063	314237	- 3174
	10	44.0	91625	90406	1219	324697	327187	- 2490
	15	49.0	92810	91589	1221	337241	339125	- 1884
	20	54.0	93753	92750	1003	348881	350183	- 1302
	25	59.0	94493	93905	588	359644	360427	- 783
	30	64.0	95364	95057	307	369428	369848	- 420

Diff - Difference

Because of this problem of error of approximation found in Regional Model Life Tables and in U.N. Model Life Tables, the improved estimates are computed for all the Regional Model Life Tables and U.N. Model Life Tables. The improved estimates in respect of West Region models of Coale and Demeny are presented in Table 3. The estimates in respect of other regional model life tables and U.N. Model Life Tables have been obtained similarly (not presented here). The interested persons may contact the author.

Conclusion

The observed difference between the originally published and the present estimates reflects the level of improvement. Over estimation is observed for $_1L_0$

TABLE 3. IMPROVED ESTIMATES - WEST REGION OF COALE DEMENY MODEL LIFE TABLES

Sex	*Levels*	${}_1L_0$	${}_1L_1$	${}_1L_2$	${}_1L_3$	${}_1L_4$	${}_4L_1$
FEMALE	1	71071	59757	54470	50832	48040	213101
	2	73684	63127	58112	54631	51941	227812
	3	76009	66186	61454	58145	55574	241360
	4	78098	68983	64541	61414	58972	253912
	5	79992	71556	67404	64465	62160	265587
	6	81719	73936	70073	67325	65161	276496
	7	83305	76147	72568	70013	67993	286722
	8	84768	78210	74912	72548	70675	296347
	9	86076	80107	77096	74933	73215	305353
	10	87386	81954	79200	77216	75636	314007
	11	88564	83662	81171	79373	77939	322147
	12	89667	85274	83040	81425	80136	329877
	13	90684	86794	84820	83392	82252	337261
	14	91596	88270	86599	85396	84436	344703
	15	92495	89647	88227	87207	86395	351477
	16	93368	90965	89778	88927	88251	357923
	17	94209	92224	91255	90562	90013	364056
	18	95016	93425	92659	92115	91685	369885
	19	95786	94570	93996	93591	93273	375432
	20	96512	95660	95270	94998	94785	380714
	21	97257	96715	96475	96310	96182	385684
	22	97952	97614	97469	97369	97293	389746
	23	98598	98415	98338	98286	98247	393287
	24	99152	99071	99038	99015	98999	396124
	25	99574	99547	99536	99529	99524	398137
MALE	]	65555	54613	49770	46501	44020	194906
	2	68651	58339	53690	50523	48103	210656
	3	71400	61724	57294	54252	51915	225188
	4	73865	64821	60626	57726	55487	238663
	5	76095	67672	63722	60975	58844	251214
	6	78125	70309	66609	64023	62009	262952
	7	79987	72761	69313	66892	65002	273969
	8	81703	75049	71853	69601	67837	284340
	9	83292	77192	74245	72163	70528	294)30
	10	84769	79205	76506	74594	73089	303395
	11	86146	81100	78646	76904	75530	312181
	12	87435	82892	80678	79103	77861	320535
	13	88632	84673	82754	81391	80316	329135
	14	89788	86355	84700	83526	82600	337183
	15	90806	87825	86395	85382	84585	344189
	16	91801	89250	88034	87175	86499	350960
	17	92767	90626	89615	88903	88343	357489
	18	93700	91954	91138	90566	90117	363776
	19	94596	93230	92602	92163	91821	369818
	20	95450	94454	94007	93697	93457	375617
	21	96332	95658	95363	95160	95004	381186
	22	97210	96765	96575	96445	96345	386131
	23	98019	97760	97652	97579	97523	390515
	24	98741	98615	98563	98529	98503	394211
	25	99322	99275	99256	99244	99234	397011

and under estimation is observed for ${}_4L_1$ in both the model life tables. The deviation of the published figures from this approximation seems to be larger at the lower levels of life expectancies. The refined approximations of person years lived at infant and childhood ages of model life table that are provided in this paper are useful for those users involved in the study of mortality, particularly of infant and childhood mortality.

Acknowledgements

The authors are extremely thankful to Dr. V. Soundararajan, Professor and Head of the Department of Statistics, Bharathiar University, Coimbatore for his valuable comments and to Dr. Thomas V. Chacko, Professor and Head of the Department of Community Medicine, P.S.G. Institute of Medical Sciences and Research, Coimbatore for his encouragements in this study. We also thank the anonimous referee whose suggestions were extremely useful.

References

Choe, M. K., 1981, Fitting the age patterns of infant and child mortality with the Weibull survival function. *Asian and Pacific Census Forum,* 7(4): 10-13. Coale, A. J. and Demeny, P., 1983, *Regional Model Life Tables and Stable Populations.* New York:
Academic Press. Hartman, M., 1980, *Infant and Child Mortality.* Lund, Sweden: Department of Statistics, University of Lund.
Keyfitz, N., 1966, Finite approximations in Demography. *Population Studies,* 19(3): 281-295. Krishnamoorthy, S., 1982, On graduating infant and child mortality with mathematical curves. *Asian and Pacific Census Forum,* 8(3) : 14-15. Mathew, A. C., 1995, Aptness of Weibull model for graduating infant and childhood mortality,
presented at the 13th Annual Conference of Indian Society for Medical Statistics in Madras,
South India. Pathak, K. B., Pandey, A. and Mishra, U.S., 1991, On estimating current levels of fertility and child mortality from the data on open birth intervals and survival status of the last child.
Janasamkhya, 9(1); 15-23. United Nations, 1982, *Model Life Tables for Developing Countries.* New York United Nations,
Department of International Economic and Social Affairs.

Appendix

Fitting the Weibull Formula for the Single Year Survivorship Probability Values Given in the Model Life Tables

In regional model life tables, l_1 and l_5 are estimated after ${}_nq_x$ is estimated. For estimating ${}_nq_x$, least square linear regressions of ${}_nq_x$ and log ${}_nq_x$ on $e°_{10}$ were used. These two regression lines intersected twice within the range of observations. To the left of the first intersection, ${}_nq_x$ values were estimated from the simple regression and to the right of the second intersection, ${}_nq_x$ values were estimated from the logarithmic regression. Between the two intersections, mean of the estimates from the two regressions is taken. Single year values of l_2, l_3 and l_4 were computed after the determination of l_1, and l_5 using the weights α_2, α_3 and α_4, and using the formula

$$l_1 = \alpha_i\, l_1 + (1 - \alpha_i)\, l_5, \quad \text{for } i = 2, 3 \text{ and } 4.$$

In U.N. Model life tables ${}_1q_x$ were estimated according to a three parameter interpolation equation

$In\ {}_1q_x = -t_1 (x + t_2)t_3$ which is claimed to perform satisfactorily. Values of t_1, t_2, and t_3 were solved such that they could reproduce the ${}_1q_0$, ${}_4q_1$ and ${}_5q_5$ values of the given model life tables.

In order to test the goodness of fit of the Weibull model, the linear form of the survivorship function

$$In\ In\ (1/l_x) = In\ \beta + \alpha\ In\ x$$

is used. Unweighted least squares method is used to estimate the parameters. The R^2 values are given in Tables A-l and A-2.

From Tables A-l and A-2, it may be infered that the fit is by and large good. However a good fit does not mean much in the light of the fact that l_2 l_3, and l_4, are in one way or other derived from l_1 and l_5 as described above. It is therefore felt to be appropriate to estimate the parameters of Weibull only from l_1, and l_5.

TABLE A -1: FITTING THE WEIBULL FORMULA FOR THE SINGLE YEAR SURVIVORSHIP PROBABILITY VALUES GIVEN IN THE COALE DEMENY MODEL LIFE TABLES FOR AGES BELOW 5 YEARS

Region	*Sex*	*Level*	R^2
West	Female	5	98.25
		15	98.16
		25	99.83
	Male	5	98.35
		15	98.10
		25	99.84
North	Female	5	99.87
		15	99.88
		25	99.30
	Male	5	99.90
		15	99.80
		25	97.95
East	Female	5	98.12
		15	97.91
		25	99.77
	Male	5	98.14
		15	97.85
		25	99.93
South	Female	5	96.31
		15	96.45
		25	99.15
	Male	5	96.63
		15	96.65
		25	99.81

TABLE A - 2: FITTING THE WEIBULL FORMULA FOR THE SINGLE YEAR SURVIVORSHIP PROBABILITY VALUES GIVEN IN THE U.N. MODEL LIFE TABLES (GENERAL PATTERN) FOR AGE BELOW 5 YEARS

Sex	*Level*	R^2
Female	10	98.81
	20	98.82
	30	98.89
	40	98.99
Male	10	98.96
	20	99.14
	30	99.40
	40	99.72