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On the Problem of Projecting the Labour Force of a Quasi-Stable Population*

1. Introduction

A population is said to be quasi-stable if one and only one of the parame-ters relat ingto the intrinsic probabilities of birth and death is independent of time. This kind of behaviour is notably observed in countries experiencing the second phase of population growth characterized by consistently decreasing mortality level coupled with almost unchanging fertility norms. Such populations are often exposed to consistent increase in population pressure. It is of considerable interest, therefore, to examine whether the age distribution of such populations under prolonged quasi-stable state becomes gradually favourable to overall increase in economically active population. It has been observed by Ypsilantis (1966) and others that for a large proportion of economically advanced and progressive countries, labour force participation rates (L.F.P.R.) consistently decline for all the age sectors save the late young and the middle age groups. It is, therefore, worthwhile to examine the overall impact of changing age distribution and the labour force participation rates (L.F.P.R.) in building up the net labour force function under a quasi-stable state. Nevertheless, methodological approach to the problem of projecting labour force, employing the techniques of quasi-stable population acquires importance in the Indian context characterized by quasi-stability espe-

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cially because the trend of labour force under such a state is a very useful item of consideration in our National Manpower Planning.

In the light of these considerations, we undertake a methodological exercise for developing an appropriate technique for the projection of labour force in India from 1981 to 2001 by taking advantages of certain recent results of Coale(1972) relating to the age distribution of a quasi-stable population.

Assumptions and the Basic Data Used

The mortality level of India has been declining since 1921. This trend is likely to be maintained till around 2000 reflecting progressive increase in the male and female expectation of life at birth. It is further assumed that the fertility level will maintain more or less a stable level, excepting for minor fluctuations till the end of this century. This assumption is based on the extent of success, achieved by the widespread 'short-term' family planning programme conducted at the state level for the reduction of fertility level in general. Such findings highlight the need of undertaking 'long-term' programmes (such as social reforms, etc.) which will naturally take long to mature.

To obtain an approximate age distribution of a population with a history of continuously declining mortality during the preceding 60 years, the following kind of data have been utilised : (i) fertility and mortality schedule for the year of projection, decided on the basis of the present fertility and mortality schedules and their likely courses in the future years, and (ii) the number of years (t) that mortality has been declining. In this regard we assumed the following values of the parameters for the years 1981 and 2001 respectively.

TABLE 1

TABLE 1

	1981	*2001*
Female expectation of life at birth (e^0_0)	52 years	62 years
Intrinsic female growth rate *(r)*	0.014	0.010
Intrinsic female birth rate (b_k)	0.033	0.026
Stable birth rate (b_s)	0.0296	0.0228
Net reproduction rate	1.685	1.344

The All India (1% sample) life table of 1971 shows that $e_0^0 = 47.1$ years for females. The above schedule of expectation of life has been obtained by increasing the female expectation of life by 5 years from 1971 onwards with the passage of every 10 years. The intrinsic growth rates (r) of females have been worked out on the basis of Ambannavar's (1974) projection of exponential growth rates and the ratio of overall growth rate and female growth rate as per the 1971 census. With the choice of two parameters e_0^0 and r the West Model tables of United Nations (1967) were utilised to get the other parameter b_s. Finally, taking the intrinsic death rate $d_k \simeq 1/e_0^0$, we equated b_k, the intrinsic birth rates, as

$$b_k \simeq \frac{1}{e_0^0} + r.$$

The figures of the net reproduction rate (NRR) for 1981 has been obtained by interpolating from Ambannavar's (1974) estimates of N.R.R. for the year 1971 and 2001 A.D. as 1.858 and 1.344 respectively.

Methodology of Projection

At the very outset, the West Model tables constructed by Coale and Demings (1967) were consulted and we attempted to identify a stable population with the expectation of life, ^{e}g = 52 and r = .014. The age distribution of such a stable population was denoted by ${}_5c(x\,;\,t)$ for the age group $[x - x + 5]$. The stable age distribution, in the next place, was adjusted for the quasi-stability caused as a result of systematic fall in the mortality level. The adjustment factor for the age group $[x - x + 5]$ was denoted by ${}_5W_l\,(x\,;\,t)$. The stable and the adjusted age distributions corresponding to the year 1981 are presented in Table 2.

The corresponding male age distribution for the same quasi-stable population (India) for the year 1981 was then obtained by employing the ratios of males and females for all the age sectors derived from the stationary male and female age distribution of the 1 % sample life table age distribution of 1971. The male age distribution is presented in Table 3.

By adopting the same technique, the quasi-stable female age distribution of 2001 has been obtained. These are presented in Table 4.

TABLE 2—THE AGE DISTRIBUTION OF THE STABLE AND THE QUASI-STABLE POPULATION (INDIA)

$(e_0^0 = 52 ; r = .014)$

Age group	*Stable age Distribution** $_5c\ (x\ ;\ t)$	*Age Distribution of the adjusted Quasi stable population (unsealed)* $_5W_1\ (x\ ;\ t)\ C\ (x\ ;\ t)$	*Age Distribution of the adjusted Quasi-stable population (Scaled)*
0-5	0.1248	0.1404	0.121
5-10	0.1115	0.1262	0.1091
10-15	0.1025	0.1166	0.1008
15-20	0.0941	0.1076	0.0931
20-25	0.0859	0.0989	0.0855
25-30	0.0780	0.0903	0.0781
30-35	0.0706	0.0822	0.0711
35-40	0.0636	0.0745	0.0644
40-45	0.0568	0.0668	0.0578
45-50	0.0502	0.0504	0.0514
50-55	0.0435	0.0518	0.0448
55-60	0.0366	0.0437	0.0378
60-65	0.0298	0.0355	0.0307
65-70	0.02 '4	0.0269	0.0233
70-75	0.0154	0.0185	00160
75-80	0.0091	0.0108	0.0093
80 and above	0.0054	0.0062	0.0054
Total	1.0000	1.1563	1.0000

"Interpolated between .010 and .015 from the West Model tables of Coale and Doming, UNO (1967).

The age distributions having been projected till 2001, we turn our attention on the reliable estimates of L.F.P.R. for estimating the proportion of labour force concentration in a particular age sector with respect to the total population. Of late, a wide range of work on this topic has come out ; but

TABLE 3-PROJECTED MALE AGE DISTRIBUTION OF THE QUASI-STABLE POPULATION OF 1981

Age group	*Male Age Distribution*
0-5	0.1182
5-10	0.1076
10-15	0.0998
15-20	0.0925
20-25	0.0854
25-30	0.0785
30-35	0.0716
35-40	00650
40-45	0.0591
45-50	0.0533
50-55	0.0463
55-60	0.0388
60-65	0.0311
65-70	0.0231
70-75	0.0155
75-80	0.0089
80 and above	0.0053
Total	1.0000

a completely exhaustive series of projection of L.F.P.R. covering all the regions of the world have been undertaken by Ypsilantis J. N. (1966). This, no doubt, gives an indication of the direction of the age sex specific L.F.P.R. over time. But since, these projections have been carried out for two broad categories of region (viz. Developed and Underdeveloped) ; these, perhaps, may not reflect the exact course of movement of L.F.P.R. of India. Nevertheless, these projections highlight broadly the trend of the L.F.P.R. over time. As against this, Ambannavar (1974) has projected the L.F.P.R. of India from 1961 onwards, taking account of the experience of some more in-

TABLE 4-PROJECTED AGE DISTRIBUTION OF MALES AND FEMALES FOR 2001 A.D.

Age group	*Stable age distribution of females*	*Adjusted female age distribution*	*Ratio of Males to Females*	*Adjusted Mate Age Distribution*
0-5	0.1032	0.0967	1.0637	0.0957
5-10	0.0965	0.0912	1.0730	0.0908
10-15	0.0911	0.0870	1.0792	0.0872
15-20	0.0860	0.0836	1.0861	0.0842
20-25	0.0808	0.0795	1.0941	0.0807
25-30	0.0757	0.0752	1.1029	0.0770
30-35	0.0709	0.0714	1.0716	0.0711
35-40	0.0661	0.0674	1.1155	0.0698
40-45	0.0614	0.0635	1.1331	0.0668
45-50	0.0564	0.0590	1.1118	0.0668
50-55	0.0510	0.0538	1.0551	0.0528
55-60	0.0450	00480	1.0160	0.0453
60-65	0.0382	0.0410	1.0154	0.0386
65-70	0,0307	0.0328	1.0295	0.0314
70-75 75-80 80 and above	0.0225 0.0)43 0.0101	0.0238 0.0162 0.0101	1.0241	0.0477
Total	1.0000	1.0000		1.0000

This ratio is on the basis of the Projected Male and Female Population of 1981 by Ambannavar (1974) Appendix 2, page 95-96.

dustrialized countries which faced similar industrial transitions during 1920-65. The changes in the L.F.P.R. during these periods were also taken into consideration, in formulating an approach for the construction of L.F.P.R. of India from 1981 onwards.

However, the plausibility of the assumptions of Ambannavar (1974) while constructing the L.F.P.R. was further screened in the light of Ypsilantis's (1966) work and then subsequently utilized for projecting the age-sex wise concentration of the proportion of economically active population. The results of Ypsilantis (1966) and Ambannavar (1974) have been placed together for comparison in Table 5 as well as for examining the validity of the basic material in this analysis.

TABLE 5 -AGE-SEX SPECIFIC L.F.P.R. FROM 1981 TO 2001 (in %)

Ypsilantis (for less Developed region)					*Ambannavar (for India)*				
Age Groups	*L. F. P. R.*				*Age Groups*	*L. F. P. R.*			
	Mate		*Female*			*Male*		*Female*	
	1980	*2000*	*1980*	*2000*		*1981*	*2001*	*1981*	*2001*
0-14	3.3	2.1	2.4	1.6	10-14	19.37	6.11	14.51	4.51
15-19	61.6	57.9	40.0	35.3	15-19	53.13	36.63	36.90	31.86
20-24	87.4	83.6	49.1	45.3	20-24	88.48	84.85	41.55	39.61
25-44	95.9	96.0	49.4	44.6	25-29	98.77	98.62	45.55	43.21
45-54	93.2	93.2	47.3	44.1	30-39	99.04	98.96	48.62	46.33
55-64	81.9	79.1	36.3	31.2	40-49	97.62	97.52	48.54	46.46
65 and above	43.2	39.3	17.7	13.8	50-59	97.94	94.49	40.77	39.04
All ages	51.9	54.6	27.7	26.5	60 and above	74.51	64.71	21.24	19.22
					Total	54.46	56.55	26.69	26.50

Since the two series of figures show some correspondence, Ambannavar's estimates have been utilized to obtain the proportion of concentration of labour force at particular age sectors with respect to the total population. The extent of concentration is exhibited in Table 6.

The estimates of labour force by age and sex were obtained by multiplying the series of figures in Table 6 with the corresponding projected population of 1981 and 2001 respectively.

TABLE 6—PROPORTION OF ECONOMICALLY ACTIVE POPULATION BY AGE AND SEX FOR 1981 AND 2001 A. D.

Age groups	*Proportion of Economically active Population*			
	1981		*200!*	
	Male	*Female*	*Male*	*Female*
10-15	0.0199	0.0146	0.0057	00039
15-20	0.0506	0.0316	0.0332	0.0266
20-25	0.0778	0.0357	0.0737	0.0315
25-30	0.0798	0.0356	0.0818	0.0325
30-40	0.1393	0.0659	0.1501	0.0643
40-50	0.1128	0.0530	0.1341	0.0569
50-60	0.0832	0,0337	0.0998	0.0397
60 and above	0.0618	0.0180	0.0821	0.0238
Total	0.6254	0.2881	0.6305	0.2792

These projections are given in Table 7 along with Ambannavar's projections (in 000) (1974) in parenthesis.

A Model for the Quasi-Stable Population

The projected figures, are in the next place graduated to eliminate biases of misreporting of the ages and distortions in the estimates of the L.F.P.R. Our objective is to evolve a theoretically intelligible model which, in the past, might have given good fit to similar sources of data.

Let $c(x\,;t)\,\delta x$ = proportion of persons belonging to the age sector $(x - x + \delta x)$ at time t in a quasi-stable population.

Then, clearly,

$$c(x\,;\,t) = \frac{c^1(x)\,W_1\,(x\,;\,t)}{\sum_x c^1(x)\,W_1\,(x\,;\,t)}, \quad (1)$$

where $c^1(x)$ is the corresponding age distribution of a stable population

TABLE 7-PROJECTION OF LABOUR FORCE IN INDIA FOR 1981 AND 2001 A.D.

Age Group	*1981*			*2001*
	Male	*Female*	*Male*	*Female*
10-15	7,054,948 (8,107)	4,788,508 (5,683)	2,968,098 (3,416)	1,880,252 (2,336)
15-20	17,938,712 (20,257)	10,364,168 (11,787)	17,287,871 (19,326)	12,824,286 (15,477)
20-25	27,581,656 (29,381)	11,708,886 (12,479)	38,376,990 (41,308)	15,186,654 (17,626)
25-30	28,290,696 (26,492)	11,676,088 (11,162)	42,594,814 (43,994)	15,668,771 (17,446)
30-40	49,384,636 (41,263)	21,613,882 (19,517)	78,159,992 (74.736)	31,000,059 (32,043)
40-50	39,989,856 (31,813)	17,382,940 (14,799)	69,828,418 (53,728)	27,432,400 (22,782)
50-60	24,496,064 (21,880)	11,052,926 (8,274)	51,967,756 (33,637)	19,140,005 (13,403)
60-above	21,909,336 (13,865)	5.903,640 (5,774)	42,751.030 (23,355)	11,474,361 (6,777)
Total	216,645,904	94,491,038	343,935,019	134,606,788

identified on the basis of the same expectation of life at birth (e_0^0) and intrinsic growth rate (r), and $W_1(x\ ;\ t)$ is the Coale's adjustment factor. Let $\pi(x\ ;\ t)$ is the L.F.P.R. in the age sector $(x - x + \delta x)$. If $B(t - x)$ represent the number of births which took place x years back from the current period t and $p(x\ ;\ t)$ represents the probability of survival upto age x under the mortality condition prevailing during $(t - x, t)$, then

$$P(t)\ c(x\ ;\ t)\ \pi(x\ ;\ t)\ \delta x = B(t - x)\ p(x;\ t)\ \pi(x\ ;\ t)\ \delta x, \qquad (2)$$

where $P(t)$ = **total population at any time** t.

Denoting $p(x;\ t)\ \pi(x\ ;\ t)$ is the net labour force function (N.L.F.F.), Biswas (1974) has shown that N.L.F.F. over ages can be graduated by a function of the form

$$f(x\,;\,t) = \frac{Aa^k}{\Gamma(k)} e^{-ax}\, x^{k-1}$$

$$0 \leqslant x < \infty. \qquad (3)$$

The parametres $A = A(t)$, $a = a(t)$, $k = k(t)$ are all time dependent. Wicksell (1931) (vide Keyfitz, 1968) employed the same graduation model for smoothing the net maternity function which is

$p(x\,;\,t)\ i\ (x\,;\,t)$ instead of $p(x\,;\,t)\,\pi(x\,;\,t)$ where $i\ (x\,;\,t)$ represents the probability of a female at age x giving a birth (female). Under the validity of such a model

$$\int_x^{x+n} c(x\,;\,t)\ \pi(x\,;\,t)\,dx = \frac{A\,a^k}{\Gamma\,(k)} \int_x^{x+n} \frac{B(t-x)}{P(t)}\, e^{-ax}\, x^{k-1}\,dx \qquad (4)$$

= proportion of labour force in the age sector

$(x - x + n)$ in relation to the total population.

Whereas the total proportion of labour force in respect of the entire population is equal to

$$\frac{A\,a^k}{\Gamma\,(k)} \ \frac{1}{P(t)} \int_0^{\infty} B(t-x)\, e^{-ax}\, x^{k-1}\,dx. \qquad (5)$$

The probability distribution of labour force in the age sector $(x - x + n)$ is given by

$$_n\alpha_x = \frac{\int_x^{x+n} B(t-x)\, e^{-ax}\, x^{k-1}\,dx}{\int_0^{\infty} B(t-x)\, e^{-ax}\, x^{k-1}\,dx}$$

$$= \frac{B(t-\xi)}{B(t-\xi')}\ \frac{\int_x^{x+n} e^{-ax}\, x^{k-1}\,dx}{\int_0^{\infty} e^{-ax}\, x^{k-1}\,dx} \qquad (6)$$

$$\simeq e^{r(\xi' - \xi)} \frac{\int_{x}^{x+n} e^{-ax} x^{k-1} dx}{\int_{0}^{\infty} e^{-ax} x^{k-1} dx}, \tag{7}$$

$\therefore$ $$B(t - \xi) \simeq B(t) e^{-r\xi},$$

and $$B(t - \xi') \simeq B(t) e^{-r\xi'},$$

$$\text{for } x < \xi < x + n \quad \text{and} \quad 0 \leqslant \xi' < \infty$$

by the mean value theorem of integral calculus. The moments of the distribution are

$$\mu_1^1 \simeq \frac{k}{a + r}, \tag{8}$$

$$\mu_2^1 \simeq \frac{k(k + 1)}{(a + r)^2}. \tag{9}$$

The estimates of k and a are obtainable from (8) and (9) by equating the sample and the theoretical moments, of corresponding orders. Also

$$\int_{w_1}^{w_2} C(x\,;t)\,\pi\,(x\,;t)\,dx = \int_{w_1}^{w_2} B(t - x) \int (x\,;t)\,dx$$

$$= A \frac{B(t - \xi)}{P(t)}, \text{ where } w_1 < \xi' < w_2$$

w_1 and w_2 here being the lower and upper bounds of the labour force participation age groups.

This gives,

$$\int_{w_1}^{w_2} C(x\,;t)\,\pi\,(x\,;t)\,dx \simeq \frac{AB\left(t - \dfrac{W_1 + W_2}{2}\right)}{P(t)}. \tag{10}$$

The L.H.S. of integral (10) was obtained by the Simpson's numerical quadrature (3/8-th rule) and A was estimated as

$$\hat{A} = \frac{P(t)}{B\left(t - \frac{W_1 + W_2}{2}\right)} \int_{W_1}^{W_2} C(x\,;t)\,\pi\,(x\,;t)\;dx. \qquad (11)$$

The estimates of k, a and A for 1981 and 2001 A.D. are presented in Table 8.

TABLE 8—ESTIMATES OF k, a AND A FOR 1981 AND 2001 A. D.

Year	Estimates of k		Estimates of a		Estimates of A	
	Male	Female	Male	Female	Male	Female
1981	6,1612	5,2609	0.1628	0.1476	70.72	34.18
2001	7,8100	7,0834	0.2035	0.1946	101.15	44.60

Given the above estimates of a, k and A, the graduated age sex specific proportions of labour force in relation to the total population were obtained by using (7) which becomes

$$_{n}\alpha_{x} \simeq e^{r}\left[\frac{w_1 + w_2}{2} - \left(x + \frac{n}{2}\right)\right]$$

$$\times \frac{\int_0^{x+n} e^{-ax}\, x^{k-1}\, dx - \int_0^{x} e^{-ax}\, x^{k-1} dx}{\int_0^{\infty} e^{-ax}\, x^{k-1}\, dx}$$

$$\simeq e^{r}\left[\frac{w_1 + w_2}{2}\right] - \left(x + \frac{n}{2}\right)\left[I_1\,(u\,;p) - I_2\,(u'\,;p)\right],$$

where

$$u = \frac{a\,(x+1)}{\sqrt{p+1}}; \quad u' = \frac{ax}{\sqrt{p+1}},$$

and $$p = k - 1,$$

and I_1 and I_2 are Pearson's incomplete Γ functions. Using Pearson's Incomplete F function table, the graduated values of the proportion of labour force for all the age sectors in relation to the total population were obtained ; then by multiplying these graduated proportions by the projected population of 1981 and 2001 A.D. respectively the graduated net labour forces by age and sex for the year 1981 and 2001 were obtained.

TABLE 9—GRADUATED LABOUR FORCE FOR THE QUASI STABLE POPULATION FOR 1981 AND 2001 A.D.

Age Groups	*1981*		*2001*	
	Male	*Female*	*Male*	*Female*
10-15	9813*	7105	7106	4000
	(8107)	(5683)	(3416)	(2336)
15-20	18201	10462	25009	12000
	(20257)	(11787)	(19326)	(15477)
20-25	26449	13347	33447	14711
	(29381)	(12479)	(41308)	(17626)
25-30	31022	14014	58695	23934
	(26492)	(11162)	(43994)	(17446)
30-40	63973	25855	100956	38119
	(41263)	(19517)	(74736)	(32043)
40-50	41912	13527	69593	27263
	(31813)	(14799)	(53728)	(22782)
50-60	17494	6736	36898	10448
	(21880)	(8274)	(33637)	(13403)
60+	7782 (13865)	3445 (5774)	12231 (23355)	4132 (6777)
Total	216646	94491	343935	134607
	(193058)	(89475)	(293500)	(127890)

*Figures under parenthesis relate to Ambannavar's estimates (1974).

Conclusion

A comparison of our age-sex specific projections for 1981 and 2001 A.D. (in Table 9) with those of Ambannavar (1974) (given under parenthesis) reveals that Ambannavar's estimates are lower than ours for both males and females. This is, largely due to inherent difference in the assumptions about behaviour of demographic parametres of fertility and mortality over time. Ambannavar (1974) has assumed the declination in the fertility level indexed by a phenomenal fall of birth rate from 34.85 to 24.09 during 1981 to 2001. Whereas our basic assumption is one of quasi-stability with minor changes in the fertility behaviour till 2001. Further, Ambannavar assumes a relatively high schedule of complete expectation of life at birth than ours. The difference in the assumptions relating to fertility rates results in relatively less persons in the working age groups in the estimates by Ambannavar ; while the difference in the assumptions about the expectation of lives at birth, might have also resulted relatively more aging in the projected population of Ambannavar. As a result, the proportion of persons in the working age groups in the total population estimated by Ambannavar is less than that of the present set of estimates.

Further, although we have basically started with the same set of labour force participation rates as Ambannavar, it is the net labour force function (in line with Wicksell) and not the set of labour force participation rates (L.F.P.R.) that we have graduated by appropriate model. Net labour force function is again a function of the product of the age distribution as well as the labour force participation rate. Since graduation has been made in respect of the function of the product, it is inappropriate to maintain that the present series is obtained merely by graduating the L.F.P.R. obtained by Ambannavar by an appropriate model. In fact, the 'Net labour force function' is radically different from Ambannavar because of the difference in the age distribution ; Ambannavar's one is based on certain assumptions relating to decline in fertility, whereas ours is based on the validity of quasi-stability.

Apart from the results of Ambannavar, the validity of our overall results can partially be corroborated by a comparison of our estimates with the recent I.L.O. estimates* for India for 1985.

Projected Labour Force

	Male	*Female*	*Total*
I. L. O. (1985)	21,9428	92217	305645
Biswas and Berry (1981-85)	21,6646	94491	311137

"Labour force projections-International Labour Office, Geneva 1971, 1965-1985, Part I, Asia (page 110).

Acknowledgements

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AUTHORS' NOTE

Because of some minor computational errors, the estimates corresponding to male labour force in Table 6, 7 and 8, have been revised after printing. The revised figures can be obtained directly from the authors.

APPENDIX I

Adjustment for obtaining the age distribution of a quasi-stable population with a history of declining mortality relative to the stable age distribution.

To obtain the approximate age distribution of a population with a history of continuous declining mortality during the preceding *t* years the following kind of data have been utilised :

(i) current fertility and mortality schedule (which identify the current stable population) ;

(ii) the number of years that mortality has been declining *(t)*;

(iii) the estimated value of *k viz.* the annual proportionate increase in fertility to which the annual increase in survival is equivalent ;

(iv) the estimated value of (5 (the slops of the line approximating the age specific mortality rate increase above the age 50).

Let $$\Delta_1 = \log_e \left(\frac{l_5}{l_0} \cdot \frac{l_{25}}{l_{30}} \right) \Big|_t ,$$

$$\Delta_2 = \log_e \left(\frac{l_5}{l_0} \cdot \frac{l_{25}}{l_{30}} \right) \Big|_{t-x} ,$$

then $$\hat{k} = \frac{\Delta_1 - \Delta_2}{x} .$$

Δ_1 and Δ_2 are obtainable from two life tables x years apart.

Estimation of $\hat{\beta}$

$$\hat{\beta} = \frac{\{{}_5m_{65} - {}_5m_{50}\}|_{t-x} - \{{}_5m_{65} - {}_5m_{50}\}|_t}{15x} ,$$

where ${}_nm_x$ refers to the central rate of mortality in the age sector $[x - x + n]$.

Again if r is the intrinsic growth rate and T is given by $e^{rT} = \log_e R_0$, where R_0 is the current net reproduction rate, and

$$m = t - \frac{3T}{4},$$

then

$$B(t-x) = B(t)\exp\left\{-r(t)\,x - k\left(1-\frac{t}{T}\right)x\right\}$$

if $x < t$,

$$= B(t)\exp\left\{-r(t)\,x - k\left(1-\frac{t}{T}\right)\right\}$$

if $x > t$,

$$\forall\ t < \frac{3T}{4}.$$

Again, if

$$t > \frac{3}{4}T,$$

then

$$B(t-x) = B(t)\exp\left\{-r(t)\,x - \left(\frac{k}{2}\right)x + \left(\frac{k}{2}T\right)x^2\right\}$$

if $x < m$,

$$= B(t)\exp\left\{-r(t)\,x - .09375\,KT + \left(-\frac{k}{2}\right)t - \left(-\frac{k}{2}\right)t^2 - k\left(1-\frac{t}{T}\right)\right.$$

if $m < x < t$,

$$= B(t)\exp\left\{-r(t)\,x - .09375\,KT - \left(\frac{k}{2}\right)t - \left(\frac{k}{2}\right)Tt^2 + \frac{ktx}{T}\right\}$$

if $x > t$.

Denoting by $R\,(x\,;\,t)$, the rates of the change and the proportion at age x in a population with a history of changing fertility at an annual rate k for t years to the population at age x in a stable population, we have, if

$t \leqslant \frac{3}{4} T,$

$$R(x\,;t) = \left(\frac{b_k}{b_s}\right) \exp\left\{-k\left(1-\frac{t}{T}\right)x\right\}$$

if $x < t$,

$$= \left(\frac{b_k}{b_s}\right) \exp\left\{-k\left(1-\frac{X}{T}\right)t\right\}$$

if $x > t$.

If $t > \frac{3}{4} T$ and $m = t - \frac{3}{4} T,$

$$R(x\,;t) = \left(\frac{b_k}{b_s}\right) \exp\left\{-\left(\frac{k}{2}\right)x + \left(\frac{k}{2}T\right)x^2\right\}$$

if $x < m$,

$$= \left(\frac{b_k}{b_s}\right) \exp\left\{-.09376\ T + \frac{k}{2}t - \frac{k}{2}t^2 - K\left(1-\frac{t}{T}\right)x\right\}$$

if $m < x < t$,

$$= \left(\frac{b_k}{b_s}\right) \exp\left\{-.09375\ KT - \frac{k}{2}t + \frac{k}{2}T\,t^2 + \frac{ktx}{T}\right\}$$

if $x > t$.

For appropriate $t \lesseqgtr \frac{3T}{4}$ and x, the adjustment factors $W(x\,;t)$ are obtained as

$$W(x\,;t) = R(x\,;t) \exp\{-(1.27\ kx)/T\},$$

and next $W_1(x\,;t)$ for the five yearly age groups as follows :

(i) $W_1(2.5\,;t) = W(2.5\,;t) \exp\{-0.63\ K\}$

(ii) $W_1(x\,;t) = W(x\,;t)$

$$x = 7.5, 12.5, \ldots, 42.5$$

where $$y = x - 45$$

and

$$s(y\,;t) = l(y\,;t)\exp\left\{-\beta\left(\frac{y^3}{2} - \frac{y^3}{3}\right)\right\}$$

$$\text{for } y \leqslant t,$$

$$= l(y\,;t)\exp\left\{-\beta\left[yt\left(\frac{y}{2} - \frac{t}{2}\right) + \frac{t^3}{6}\right]\right\}$$

$$\text{for } y > t.$$

$W_1(x\,;t)$ are thus the set of multipliers that may be applied to the stable age distribution to give the number of persons in each group in a population with history of declining mortality.

The proportionate age distribution by five year age intervals of this population is

$$\frac{W_1(x)\,c\,(x\,;t)}{\sum_x W_1(x)\,c\,(x\,;t)}\,.$$

APPENDIX II

Assumptions Regarding the Future Course of Labour Force Rate, 1961-2011

Ambannavar J. P. (1974)

Before stating the assumptions, it is important to state that there are no data on the past trends in the age-specific labour force rates which cculd have served as a guide to assumptions regarding the future course. The 1961 and 1971 census tabulated economic activity data by age but, for the reasons stated in the next sub-section, the data are not at all comparable. Hence, it is inevitable to take recourse to guesses and sometimes to the experience of other countries.

Assumptions Regarding Changes in LFRs, 1961-2011 LF Rates in the 10-14 Age Group

In this age group, the 1961 LFRs for both males and females in rural as well as urban areas, are assumed to linearly decline to zero by 2011 on account of rising participation in schools.

LF Rates in the 15-19 Age Group

For males, the LFR is assumed to linearly decline "by 50 per cent in rural areas and 75 per cent in urban areas. This gives an approximately 60 per cent decline in the rate for all artas. This assumption is based on the Japanese experience during 1920-65 when the LF rate in that country declined by 55 per cent. In assuming a nearly similar percentage change for India during 1961-2011, there is no implicit assumption that India will experience a similar socio-economic change during 1961-2011 as Japan did during 1920-65, but that it may nevertheless achieve substantial progress in school enrolment in this age group. In the past, growth has been much faster in the field of education than in any other socio-economic aspects, and a similar imbalance between growth of education and overall economic development may be expected in the future also.

The LF rates for females in rural and urban areas are assumed to remain constant since a large proportion of girls in this age group do not participate either in schooling or in economic activity. In 1961, only 4 per cent of the girls in this age group had enrolled in schools, 36 per cent returned themselves as economically active, and 60 per cent as "others". Since 70 per cent of the girls in this age were married in 1961, a large proportion of the girls in the "active" and "other" categories perhaps had child-bearing and child-care functions. With the possibility of a rise in the age at marriage an increasing proportion of girls in this age group will come to be spared of marriage and the consequent functions of child bearing and care. The rise in school participation will draw upon this group, thus leaving the LFR unaffected during the period under consideration.

LF Rates in the 20-24 Age Group

In this age group the LFR for all areas will linearly decline by 10 per cent for males and the entire decline will come about as a result of changes in the LF rate in urban areas only. The percentage decline assumed here is based on the Japanese experience during 1920-65 when the LF rate decreased by 9 per cent. The assumed decline will come about as a result of growth of higher education. Since higher education is concentrated almost entirely in urban areas, the decline in LF rate will be wholly confined to urban areas.

For females, it is assumed that the LF rates in rural and urban areas will continue to remain the same.

LF Rates in the 60 and Over Age Group

Due to increasing longevity of life, an increasing number of persons come to survive beyond age 70, such that the proportions of persons in the 70 and over age group to the persons in the 60 and over age group will rise. Since activity rates at extreme old ages are very low, the LF rate for age group 60 and over will decline. Assuming that the LF rate in the age groups 60-64 and 65-69 will continue as it is and that in the 70 and over age group will linearly decline by 50 per cent during 1961-2011, the LF rate in the age group 60 and over was estimated to decline in the manner indi cated by the index below :

INDEX OF LF RATE IN AGE 60 AND OVER

	1961	1971	1976	1981	1986	199i	1996	2007	2006	2007	2011
Male	100	96	95	94	93	92	90	87	85	85	84
Female	100	98	97	97	96	95	93	92	89	89	86

The LF rates for both males and females in rural as well as urban areas are assumed to decline according to the index given above.

LF Rates in other Age Groups

LF rates in age groups 25-29, 30-39, 40-49 and 50-59 for both males and females in urban as well as rural areas are assumed to remain constant at 1961 levels.

Assumption Regarding "Main Worker" Rates (1971 Criterion)

It is widely recognised that the concepts used in the measurement of the working force differed greatly between the 1961 and 1971 censuses. For the purpose of understanding the concept of worker in the two censuses, the population may be classified into four broad categories :

(i) Persons who perform both economic and non-economic activities but spend most of their active time in the former; in this category fall most of the adult males; for convenience, these are called "main workers".

(ii) Persons who perform both economic and non-economic activities but spend most of their active time in the latter; in this category fall some of the women, students, youngsters, etc. These are called "secondary workers".

(iii) Persons who perform only non-economic activities such as housekeeping, schooling etc.; in this category fall some of the housewives, students, etc.

(iv) Persons who do not perform either economic or non-economic activities; to this category belong infants, very young, disabled, etc.

It is universal to consider persons in the first category as workers and those in the third and fourth categories as non-workers, as was done in both censuses. However, the two censuses differed significantly in the treatment of the second category of persons. In the 1961 census, they were included among workers, along with, but not separable from, the first category of persons. These workers (i.e. category (i) and (ii) plus the unemployed persons) constituted labour force which forms the basis of labour force rates in 1961 presented in Table 3.4 (Ambannavar 1974). In the 1971 census, on the other hand, they were included among non-workers but they could be distinguished from other non-workers on the basis of their secondary activity being recorded as economic activity. This difference in the treatment is not of great importance as long as it is possible to identify this category and add it up with the first category of persons. But the problem is that the 1971 census concentrates mainly on main workers who may be considered full-time workers. Rates estimated on the basis of main workers are called "main worker" rates. The difference between the LF rates as estimated for the year 1971 and main worker rates of 1971, may be considered as constituting secondary worker rates. Table 3.6 (Ambannavar 1974) presents LF rates and main worker rates in 1971.