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Estimation of Birth Rate from a Prospective Sample (A Macro-Simulation Study)

Introduction

A prospective sample is defined as one in which a sample is drawn from a population at a given point of time and then the sample members are followed over time. Usually standard sampling technique is used to draw the original sample and follow up of the sample members is done at equi-distant time intervals. Such studies are fairly common in Demography. Evaluation of family planning performance, estimation of post-partum amenorrhoea, population laboratory etc. are some examples of follow-up studies.

As we follow a sample, the members get depleted due to death, migration and various other reasons. These dropouts affect members in different age groups differently. Similarly, members in the sample from different income, profession and education groups are affected differently. Even in the same category, variations may occur between rural and urban areas. As these depletions do not occur randomly, the representative character of the sample is compromised and the residual samples at later dates are not random samples from the population on these dates. Further, if there is time trend in the factors responsible for dropout the situation becomes more complicated from statistical point of view. In a time series it is difficult to construct test functions unless some simplifying assumptions are made. In this study the aim is to estimate the true population birth rate from the residual sample. The

method can be used to estimate other rates also and coverage can be a region or the country as a whole.

Methodology

It is obvious that we need a set of multipliers for marking up the number of women and estimated number of births after a given number of years from the starting year. For the purpose of illustration we will consider a 20 percent simple random sample drawn from all India female population in the ages 10-49 in 1961. This sample is followed up for five years. Macro-simulation approach is adopted to estimate the number of women in the sample in 1962, 1963 . . . 1966. Let ${}_{61}P_{10}, {}_{61}P_{11}, \ldots, {}_{61}P_{49}$ be the number of women aged 10, 11, . . . 49 in 1961. Let ${}_{61}X_{10}, {}_{61}X_{11}, \ldots {}_{61}X_{49}$ be the corresponding sample members. So

$$E({}_{61}X_i) = n \frac{{}_{61}P_i}{P}, \tag{1}$$

where E stands for expectation, n is the sample size, $n = (.20)P$ and

$$P = \sum_{i=10}^{49} {}_{61}P_i.$$

It can be seen that

$$V({}_{61}X_i) = n \frac{{}_{61}P_i}{P} \left(1 - \frac{{}_{61}P_i}{P} \right). \tag{2}$$

Thus the actual number of women in the sample in 1961 at various ages 10, 11, 12, . . . will have the mean and variance as given in (1) and (2). ${}_{61}P_i \pm 3\sqrt{V({}_{61}X_i)}$ give the possible range for ${}_{61}X_i$. From this the 99 percent confidence interval for the sample estimate of 1961 birth rate can be calculated. Using ${}_{61}P_i$ from 1961 census — Age tables — India[5] and age specific fertility rates as given by Sinha [6] the crude birth rate is found to be 43.60 and the confidence interval, to be $\pm$ .16. Thus in repeated sampling from the population with the given age distribution and age specific fertility rates it is almost certain that the estimated birth rate will lie in the range 43.44 and 43.76. Single year survival ratios for females for the years 1961, 1962 . . . 1966 are

interpolated from Sinha[7]. Migration rates are taken from Vaidyanathan [8]. The survival ratios reflect a decrease in mortality over the years. Age specific fertility rates show a slight decrease in total fertility in the same period. Age specific female migration rates are assumed to be constant over the years.

Loss of members in the sample are assumed to occur due to (i) deaths, (ii) migration and (iii) other causes. But, for the population as a whole migration and other causes can be assumed to have no significant effect. Due to improvement in mortality, the numbers in individual ages show increases. Drop-outs from sample due to other causes is assumed to be related to fertility performance in the preceding year. If a women had a birth in the preceding year she is given an average chance of 10 percent with standard deviation of .05 for dropping out from the sample. Those who did not give a birth in the preceding year are assumed to be unaffected by this chance of dropout. Conditions for dropout and the extent of such dropouts can, of course, be varied. The reason for dropout due to child birth assumed here is that in a prospective survey of family planning method users, a woman who had an accidental or planned pregnancy in the previous year is more likely to be out of the sample in the following years. For migrants and non-migrants same age specific fertility rates are used for computation of births in any year. This may not be appropriate as the migrants are likely to have relatively lower fertility. Table 1 shows the number of women by age in the population and sample for each year. The columns in the table have been built up as follows :

$$
\begin{aligned}
&{}_{61}P_{10} = {}_{61}P_{9}\ {}_{61}S_{9}, \quad {}_{63}P_{10} = {}_{61}S_{8}\ {}_{62}S_{9}\ {}_{61}P_{8} \cdots \\
&{}_{66}P_{10} = {}_{61}S_{5}\ {}_{62}S_{6}\ {}_{63}S_{7}\ {}_{64}S_{8}\ {}_{65}S_{9}\ {}_{61}P_{5}. \quad \text{In general,} \\
&{}_{66}P_{j} = {}_{61}S_{j-5}\ {}_{62}S_{j-4}\ {}_{63}S_{j-3}\ {}_{64}S_{j-2}\ {}_{65}S_{j-1}\ {}_{61}P_{j-5}. \qquad (3)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{62}X_{10} = 0, \quad {}_{62}X_{11} = ({}_{61}S_{10}\ \bar{M}_{11} - {}_{61}f_{10}\alpha)\ {}_{61}X_{10} \\
&{}_{63}X_{10} = 0, \quad {}_{63}X_{11} = 0, \quad {}_{63}X_{12} = ({}_{62}S_{11}\ \bar{M}_{12} - {}_{62}f_{11}\ \alpha) \\
&\qquad\qquad\qquad\qquad\qquad\qquad ({}_{61}S_{10}\ \bar{M}_{11} - {}_{61}f_{10}\ \alpha)\ {}_{61}X_{10}.
\end{aligned}
$$

Similarly

$$
\begin{aligned}
&{}_{66}X_{10} = 0, \ {}_{66}X_{11} = 0, \ {}_{66}X_{12} = 0, \ {}_{66}X_{13} = 0, \ {}_{66}X_{14} = 0 \ \text{and} \\
&{}_{66}X_{15} = ({}_{65}S_{14}\ \bar{M}_{15} - {}_{65}f_{14}\ \alpha)\ ({}_{64}S_{13}\ \bar{M}_{14} - {}_{64}f_{13}\ \alpha) \cdots \\
&\qquad\qquad\qquad\qquad ({}_{61}S_{10}\ \bar{M}_{11} - {}_{61}f_{10}\ \alpha)\ {}_{61}X_{10}
\end{aligned}
$$

In general

$$_{66}X_j = (_{65}S_{j-1}\ \overline{M}_j - {}_{65}f_{j-1}\,\alpha)\ (_{64}S_{j-2}\ \overline{M}_{j-1} - {}_{64}f_{j-2}\,\alpha) \ldots (_{61}S_{j-5}\ \overline{M}_{j-4} - {}_{61}f_{j-5}\,\alpha)\ {}_{61}X_{j-5} \quad (4)$$

where $_iP_j$ and $_iX_j$ [$i = 61, 62, \ldots 65,\ \ j = 10, 11, \ldots 49$] stand for the actual female population and females in the sample respectively in the year i at age j. $_iS_j$ stand for survival ratio in year i at age j; $\overline{M}_j = (i$ — age specific female internal migration rate at age j); $_if_j$ = age specific fertility rate in year i at age j); and α is a random variable in the range 0 — 1 determined from a normal population with mean 0.10 and standard deviation .05.

It can be seen that $_iX_j < {}_iP_j$ for all i and j. This is natural. So $E(_iX_j)$ will always be less than $_iP_j$ and the difference will increase as i increases. Estimated births from the residual sample (net number of women ramaining in the sample) will be $\sum_j {}_iX_j\ {}_if_j$ and the sample estimate of birth rate in the year i will be

$$(B.R)_i = \frac{\sum {}_iX_j\ {}_if_j}{\sum {}_iX_j} \times K_i, \quad (5)$$

where $$K_i = \frac{\text{the number of women in ages } j \text{ to 49 in the year } i}{\text{Total population in the year } i}.$$

In this work we assume that proportion of females in ages j to 49 to total population in the years 1962 to 1966 will be the same as the corresponding proportions in 1961. As the number of years considered is 5 and as there has been no radical change in fertility or mortality, the use of K_i based solely on 1961 population is perhaps valid. If the period of follow up is longer, age composition of the population will change, accordingly K_i should also be changed.

From (3) and (4) it can be seen that if $_{66}X_j$ is multiplied by the factor

$$_{66}W_j = \frac{_{61}S_{j-5}\ {}_{62}S_{j-4}\ {}_{64}S_{j-3}\ {}_{64}S_{j-2}\ {}_{65}S_{j-1}}{(_{65}S_{j-1}\ \overline{M}_j - {}_{65}f_{j-1}\alpha)\ (_{64}S_{j-2}\ \overline{M}_{j-1} - {}_{64}f_{j-2}\,\alpha) \ldots (_{61}S_{j-5}\ \overline{M}_{j-4} - {}_{61}f_{j-5}\alpha)} \quad (7)$$

then the quantities $_{66}W_j\ {}_{66}X_j$ will be unbiased estimate of $_{66}P_j$. Now calcula-

tion of ${}_{66}W_j$ for $j = 15$ to 45 is not easy. Some sort of approximation has to be used. One apparent approximation is to replace individual S_j, $\overline{M}_j$ and f_j by the group averages in 15-19, 20-24, 25-29, . . . 44-49.

For example

$$ {}_{66}W_{25} \sim \left\{ \bar{\mu}_{25} \left[1 - \frac{\bar{f}_{25}\,\alpha}{\bar{S}_{25}\,\bar{\mu}_{25}} \right] \right\}^{-5}, \tag{8} $$

where

$$ \bar{\mu}_{25} = \frac{\overline{M}_{25} + \overline{M}_{24} + \overline{M}_{23} + \overline{M}_{22} + \overline{M}_{21}}{5} $$

$$ \bar{f}_{25} = \frac{{}_{65}f_{24} + {}_{64}f_{23} + {}_{63}f_{22} + {}_{62}f_{21} + {}_{61}f_{20}}{5} $$

$$ \bar{S}_{25} = \frac{{}_{65}S_{24} + {}_{64}S_{23} + {}_{63}S_{22} + {}_{62}S_{21} + {}_{61}S_{20}}{5}. $$

Table 1 gives the seven ${}_{66}W_j$ for the age groups 15-19 to 44-49. It can also be seen from the table that when the estimated number of births and women are weighted by the weights ${}_{66}W_j$ the estimated birth rate from the net sample females exactly reproduce the actual population birth rate in 1966. The agreement is fantastic, but as has been pointed out earlier, the variance of birth rate is rather small; the weights should give a birth rate which is close to the true population birth rate.

An interesting variation is to give specific time trend to ${}_iS_j$, ${}_iM_j$, ${}_if_j$. For example, we may assume

$$ {}_iS_j = {}_iS_{j-1} - a_{(j-15)}, \tag{10} $$

$$ {}_iS_j = {}_{i+1}S_j - b_{(j-15)}. \tag{11} $$

(10) states that in the same year as age increases survival ratios linearly decrease. (11) states that for the same age, survival ratios increase as we pass from one calendar year to the next. In reality (11) is perhaps not correct for the earlier and higher ages. For the middle ages 20-35 (11) may hold. In the earlier ages ${}_iS_j$ may increase till we reach a maximum and then slowly decrease. The same pattern is likely to hold for higher ages. This means that for these age groups (11) should be replaced by a 2nd order equation.

$_iM_j$ behaves differently in different age groups, [for detailed discussion on $_iM_j$— its variation with age, income etc. refer to Vaidyanathan *(op. cit)]*. Similarly $_if_j$ can be given a linear decreasing pattern. These variations can be studied through a simulation model.

References

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7. Sinha, U. P., *op. cit.,* Appendix-I.
8. Vaidyanathan, K. *E., Internal Migration in India 1951-1961,* Table 3, p. 8, Demographic Training aud Research Centre, Chembur, Bombay.

	Population 1961 (females] in '00	Population 1962 in '00	Sample 1961 actual	Survivors of sample in 1962	Sample adjusted for migra-tion 1962	Loss due to other causes 1962	Net females in sample 1962	Population 1963 in '00
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	(8)
5	60833	62246.35						
6	58749	60274.55						61687.38
7	56838	58355.97						59879.75
8	55005	56487.31						58004.67
9	53243	54686.52						56168.72
10	51521	52952.29	876875					54396.13
11	49849	51254.12	851822	872333	841278		841278	52685.94
12	48258	49596.26	1102357	847503	817671		817671	51001.95
13	46772	48011.88	952036	1096735	1059336	484	1058852	49351.25
14	45367	46529.25	1052250	947095	914799	1348	913451	47769.90
15	44045	45125.19	1155404	1046642	1017127	2328	1014799	46288.23
16	42805	43802.31	602819	1149038	1116176	11499	1104677	44883.32
17	41636	42561.01	778642	599383	583020	9662	573358	43558.33
18	40522	41389.51	929346	774032	752746	15445	737301	42314.16
19	39469	40272.79	703289	923630	899246	20785	878461	41139.51
20	38514	39216.40	653343	698788	680899	19082	661817	40019.88
21	37672	38257.11	929757	648985	632436	22968	609468	38959.14
22	36904	37409.82	753857	923286	899834	14665	885169	37995.05
23	36163	36636.45	653343	748392	729458	35620	692938	37143.07
24	35440	35892.14	703600	648449	632108	24359	607749	36366.44
25	34748	35167.11	677375	698182	680518	25869	654649	35620.08
26	34059	34473.84	752639	672030	654759	14373	640386	34894.21
27	33357	33783.46	677375	746550	727438	10830	716608	34199.43
28	32669	33079.14	602111	671732	654603	20742	633861	33507.11

29	32002	32388.05	627199	596933	581592	23009	558583	32799.62
30	31229	31716.54	759650	621604	605567	7950	597617	32104.33
31	30272	30939.51	578688	722886	704163	22046	682117	31427.92
32	29199	29980.18	603848	573109	558552	16125	542427	30647.13
33	28147	28906.72	654169	597803	582380	14159	568221	29685.77
34	27084	27854.27	652245	647366	630988	1787	629201	28612.45
35	26066	26791.76	374691	348444	343113	4310	338803	27560.13
36	25123	25774.06	724402	370494	365159	7714	357445	26498.39
37	24219	24831.07	499588	715984	707750	6241	701509	25481.27
38	23324	23927.16	424649	493568	488484	8557	479927	24538.56
39	22420	23032.22	399670	419337	415050	4687	410373	23635.01
40	21583	22129.21	402700	394486	390502	472	390030	22740.86
41	20829	21292.28	478206	397276	393542	2595	390947	21838.65
42	20111	20537.60	427869	471516	467131	2216	464195	21002.07
43	19413	19819.39	377531	421665	417744	1943	415801	20247.61
44	18739	19122.78	327194	371887	368466	782	367684	19531.02
45	18076	18451.54	469866	322175	319211	1709	317402	18837.28
46	17407	17792.21	449866	442803	439083	1469	437614	18169.60
47	16727	17127.10	399880	442632	439312	1959	437361	17513.58
48	16079	16449.16	249925	393238	390328	740	389588	16849.98
49	15437	15799.22	124963	245576	243783	393	243390	16170.35
B.R.	43,60	4309	43.68	42.87	42.84	—	42.47	42.94

TABLE 1 *(Contd.)*

	Survivors of net 1962 sample in 1963	Sample adjusted for migration 1963	Loss due to other causes 1963	Net females in sample 1963	Population in 1964 (in '00)	Survivors of sample in 1964	Sample adjusted for migration 1964	Loss due to other causes 1964	Net females in sample 1964
	(9)	*(10)*	(11)	*(12)*	*(13}*	*(I4)*	*(15)*	*(16)*	*(17)*
5									
6									
7					61291,96				
8					59527.66				
9					57685.64				
10					55878.33				
11					54130.68				
12	837139	807672		807672	52434.63				
13	813632	785887	278	785699	50757.65	803803	776393	0	776393
14	1053515	1017590	1632	1015958	49109.92	781767	755109	771	754338
15	908719	883093	1629	881464	47529.14	1010838	982332	1481	980851
16	1009360	980492	6086	974406	46046.14	876854	851776	5712	846063
17	1098524	1068534	5788	1062746	44639.15	969105	942648	16938	925710
18	570032	554356	2172	552184	43310.92	1056710	1027650	17929	1009721
19	732848	713501	15213	698288	42063.66	548915	534424	3571	530853
20	872944	850597	12639	837958	40885.68	693980	676214	24543	651671
21	657475	640709	4284	636425	39762.15	832562	811332	23730	787602
22	605293	589919	15070	574849	38696.56	632135	616079	24539	591540
23	878858	856623	18831	837792	37728.70	570819	556377	14026	542351
24	687831	670498	35019	635479	36873.41	831710	810751	40685	770066
25	603142	587883	29615	558268	36095.51	630745	614787	12224	602563

26	649569	632875	10781	622094	35348.30	554008	539770	34448	505322
27	635288	619025	14343	604682	34620.99	617223	601422	17017	584405
28	710746	692622	29539	663083	33924.12	599814	584519	22532	560987
29	628505	612352	13543	598809	33229.00	657579	646174	27197	618977
30	553690	539405	11983	527432	32517.54	593659	583420	15184	568236
31	592179	576842	20364	556478	31817.64	522712	513814	0	513814
32	675671	658509	13038	645471	31136.58	551319	542065	13774	528291
33	537100	523243	10354	512889	30352.30	639262	629076	14883	614193
34	562437	548207	3926	544281	29389.51	507770	499656	11596	488060
35	622557	613032	20137	592895	28316.60	538653	530511	5790	524721
36	335693	330268	10060	320199	27264.96	586545	583824	15645	568178
37	353384	349320	2107	347213	26204.26	316645	315588	2573	313015
38	693245	686105	13642	672463	25188.24	343220	313004	5946	307058
39	474067	469232	4607	464625	24246.06	664447	665537	3648	661889
40	405182	401093	5148	395942	23579.70	458891	459886	4149	455737
41	384909	381291	3753	377538	22449.78	390874	391943	2266	389677
42	385618	382032	3707	378325	91548.62	372521	393989	1557	372432
43	458350	454087	1424	452663	20713.29	373123	374806	222	374584
44	409751	405951	1040	404941	19906.70	446249	448453	2644	445809
45	362194	358862	2634	356228	19247.04	399053	401216	1034	300182
46	312651	310025	759	309266	18556.98	350927	352951	0	352951
47	430761	427530	1804	425726	17892.51	304550	306668	1074	305594
48	430284	427100	446	426654	17237.56	419016	422533	804	421729
49	382985	380188	507	379681	16571.79	419610	423497	88	423409
B. R.	42.33	42.26	—	41.98	42.72	42.08	41.93	—	41.58

TABLE 1 *(Contd.)*

Age	Population in 1965 in '00	Survivors of sample in 1965	Sample adjusted for migration 1965	Loss due to other causes 1965	Net females in sample 1965	Population 1966 in '00	Survivors of sample in 1966	Sample adjusted for mig-ration	Loss due to other causes 1966	Net females in sam-ple
	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
8	60939.53									
9	59208.59					60621.42				
10	57396.06					58919.06				
11	55613.47					57132.04				
12	53880.60					55364.32				
13	52191.33					53638.14				
14	50517.06	772713	746363	1245	745118	51951.25				
15	48869.77	750649	729481	2472	727009	50276.60	741571	720659	876	719783
16	47287.22	975858	947948	0	947948	48627.38	723403	702714	4982	697732
17	45801.17	841562	818587	5903	812684	47041.33	943187	917438	7271	910167
18	44390.51	920554	895239	10949	884290	45551.55	808255	786028	10281	775747
19	43056.68	1003794	977294	35489	941805	44137.04	879241	856029	12427	843602
20	41808.75	527636	514128	7913	506215	42800.06	936192	912225	25321	886904
21	40626.87	647546	631034	13609	617425	41548.70	503066	490238	9145	481093
22	39498.92	782388	762515	28258	734257	40362.39	613406	597825	6662	591163
23	38429.55	487458	572595	16689	555906	39230.72	729271	710820	10861	699959
24	37459.32	538479	524909	9724	515185	38159.77	552004	538093	12102	525991
25	36603.13	764421	745081	34669	710412	37188.86	511465	498525	12894	485631
26	35824.43	598038	582668	21626	561042	36332.63	705162	687039	5532	681507
27	35076.12	501431	488594	23059	465535	35553.24	556795	542541	13093	529448
28	34347.14	579782	564998	12299	552699	34802.23	461913	450134	17474	432660
29	33647.30	556409	542109	21194	520915	34071.68	548266	534176	19705	514471
30	32948.21	613747	597912	16574	581338	33368.03	516591	503263	17203	486060

31	32232.36	563252	548664	8463	540201	32664.52	576333	561406	12774	548632
32	31528.10	509138	496206	13042	483164	31944.85	535312	521783	16833	504950
33	30842.96	523309	509808	3180	506628	31236.78	478700	466350	3236	463114
34	30055.76	608192	592805	14701	578104	30547.79	501780	489085	14024	475061
35	29092.09	483121	475729	1555	474174	29757.91	572375	563618	10076	553542
36	28019.84	519222	511745	14611	497134	28793.90	469314	462556	1617	460939
37	26969.14	562013	555550	12234	543316	27722.55	491859	486203	6248	479955
38	25909.72	309497	306309	2666	303643	26673.02	537350	531815	9913	521902
39	24895.30	303487	300391	0	300391	24603.03	300194	297132	2938	294194
40	23954.14	653920	647315	7892	639423	24603.03	296864	293866	1815	292051
41	23285.66	450054	445823	0	445823	23662.86	631648	685710	7261	618449
42	22159.28	384634	381057	1382	379675	22992.26	440206	436112	1640	434472
43	21260.09	367445	364028	539	363489	21870.54	374728	371243	3612	367631
44	20427.65	369418	366019	1936	364083	20974.78	358611	355312	1244	354068
45	19678.26	439501	435458	2315	433143	20146.36	359070	355766	1231	354535
46	18968.34	394387	391074	1021	390053	19400.80	427036	423449	1196	422253
47	18281.41	347710	345102	15	345087	18694.06	384413	381530	1247	380283
48	17617.86	300903	298678	650	298026	18008.10	339928	337412	569	336843
49	16960.38	414948	438722	339	438383	17341.79	293356	291214	340	290874
B.R	42.59	41.79	41.63	-	41.34	42.48	41.49	41.36	—	41.21

Inflation factor (W)

15-19	1.1998	35-39	1.2050
20-24	1.2866	40-44	1.1045
25-29	1.3306	45-49	1.1064
30-34		1.2977	

When the inflation factors are used we have from the 1966 sample

Estimate births = 4,161,304

Estimated women = 22,608,322

and the estimated birth rate is 42.48 which is exactly equal to the population birth rate in 1966.