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A Study of Internal Migration India: An Application Markov Chains

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Abstract

Migration has always been a very important component of growth of a population. The growth of a population though is studied "closed" to migration. For any developing nation the migrants are important for economical and structural development as they provide human resource to any region. Initially it was considered that the distance between the place of origin and place of destination is the most important factor affecting migration and so the models were developed according. But as we study it further we have realized that migration tends to change with gender and age at major. Migration is assumed to be a Markov process where probability of migration from one place to another can be studied as probability of transition from one to another state and any future state can state depends only on current state where different geographical areas are considered as different states of a Markov Chain.

In this paper we have studied the mechanism of internal migration with the help Markov Chain analysis among various states of India. The Markov transition matrix between states in two consecutive periods is parameterized and estimated using maximum likelihood estimation. We have found that as the past researches have also stated, migration mechanism can be successfully studied with Markov Chain analysis. Under this study we have found that migration flow is expected to follow a reverse trend in the near future. The reverse trend is expected to occur from the areas with particularly heavy in-migration in the present such as Delhi and Maharashtra.

Introduction

One of the most important single demographic facts about a population is its rate of growth (Donald J. Bogue). The growth of a population depends on the status of fertility, mortality and migration of people. Though the natural growth of a population is measured through the count of births and deaths in a population, migration plays equally important role in composition of a population. It is a well-known fact that the contribution of migration has continuously been increasing in the construction of physical and cultural structure of a population. The occurrence of a migration depends on both personal attributes (systematic factors) and chance. Migration is defined as a move from one migration defining area to another, usually crossing administrative boundaries made during a given migration interval and involving a change of residence (UN 1993). Change in migration level can bring major changes in socio-economic and cultural composition of a population. Over time, a population may be changed or transformed as people realize their intentions to enter or leave an area (Morrison, et. al, 2004, in Siegel and Swanson 2004). Both internal and international migrations are considered to be important factors while studying the fundamental structure of population. But at the same time migration is considered to be complicated topic for researchers. Internal migration plays a vital role in bringing change in not just the structure but eventually changing the socio-economic state of a region. It, therefore, becomes necessary to study migration and its differentials in details. Over the period of time, different researches have been carried on migration, most vital of which were done in mid 1900s. Ravenstein (1885-89), E.S. Lee (1966), Sjaastad (1962), A. Rogers were among the few researchers how studied migration differentials and had developed various theories leading to important conclusions about population growth. Based on various assumption and these differentials various migration models were developed further to understand migration e.g. Zipf's model (1940) , Stouffer's model (1940), Jhonson model (1952), Rogers Model (1967) etc.

Migration is clearly patterned non-random phenomenon that is subjected to scientific explanation and therefore, perhaps ultimately may be forecast with a reasonable degree of accuracy (Andrei Rogers, 2010). In developing probability models of migration and migrants, the occurrence of an event (migration) is assumed to be the result of an underlying random mechanism. In the recent past, probability models have proven to be more realistic and applicable in many situations than deterministic models. In past many years, various quantitative models based on internal migration have received importance in particular areas of demography and social sciences. Data have been collected all over the world and probability models have been proposed to study the empirical aspects of internal migration. But the absence of abundant and reliable information on migration gives rise to the need of more efficient statistical models to study migration in a geographical area. The growing interest in quantitative analysis of migration phenomenon has been the emergence of Markov chain theory as a methodological tool for analyzing social, industrial and geographic mobility. Sociologists and economists have pursued theoretical models for studying human mobility. There are different tools developed for the detailed study of migration but it becomes very complicated to explain since migration itself is a very complicated factor. The occurrence of the event, migration, is a random process. For this purpose Markov chain models have been developed and being used. In these models, various geographical locations are the states in Markov chains, and the transition probabilities are either empirically estimated or assumed to possess certain properties. In these models, various geographical locations are the states in Markov chains, and the transition probabilities are either empirically estimated or assumed to possess certain properties (McGinnis, 1968; Bartholomew, 1967; and Henry, McGinnis and Tegtmeyer, 1971).

Internal Migration in India

The following table gives the absolute figure (in million) of lifetime migrants based on the place of last residence criterion. In 1971 census, 160 million comprising of 50 million males and 110 million females, were termed migrants on the basis of place of last residence. This constitutes 30.6 per cent of the total population of the country.

Internal Migrants by Sex, India 1971-2011

Year	Lifetime Migrants (In million)			Percentage of Migrants		
	Persons	Males	Females	Persons	Males	Females
1971	159.6	49.6	110.0	30.6	19.0	43.1
1981	201.6	59.2	142.4	30.3	17.6	43.9
1991	225.9	61.1	164.8	27.4	14.6	41.2
2001	309.4	90.7	218.7	30.6	17.5	44.6
2011	453.6	140.9	312.7	37.5	11.6	25.8

In term of total volume of migration, the figure has increased to 201 million in 1981, 226 million in 1991 and 453.6 million in 2011. The percentages of migrants to total population however declined to 30.3 per cent in 1981 and further to 27.4 per cent in 1991. It has however increased to 30.6 per cent in 2001 and 37.5 per cent in 2011. Sex wise differences are very prominent in Indian migration data. It has been observed that majority of migrants are females.

Growth of Internal Migrants by Sex, India 1971-2001.

Year	Lifetime Migrants (%)		
	Persons	Males	Females
1971-81	26.3	19.43	29.4
1981-91	12.04	3.21	15.72
1981-91	36.96	48.33	32.75
1971-2001	93.82	82.83	98.78

The above table shows the growth of migrants among lifetime migrants. It is evident that there is a steep increase in the growth of migrants in 2001 with males achieving about 50 per cent increase over that of 1991 figure. In the present study, we have tried to analyze the mechanism and to find a Markov Chain model for the internal migration among various states in India. We have tried to study the properties of the transition probability matrices, to find the stationary probability, and behavior of the mechanism of the migration in the year 2001 due to limitation of data. The data used to study the level and trend of migration is taken from Census of India, 1971-2001. Migration can be measured either as events or transitions. Besides, we have studied the properties of the behavior of transitions probability matrices with respect to migration. We have used Inter-state migration data for five states with heavy migration flow. These five states are Bihar, Uttar Pradesh, Madhya Pradesh, Delhi and Maharashtra. Various states are considered as the transition states of Markov Chains assuming the probability of occurrence of migration at $(n+1)^{th}$ state depends only on the migration at n^{th} state irrespective of migration in past time periods. Stochastic modeling can further be used to control the growth or decline of a population. The Markov transition matrix between states in two consecutive periods is parameterized and estimated using maximum likelihood estimation.

Markov Chain Analysis

The growth of population is considered to be a stochastic procedure. In this section our main interest is the behavior of Markov chains as they are framed for migration phenomenon. Consider the data from observation in finite Markov chain with states (1, 2, ..., k) until $m_{..}$ transitions have taken place. Let m_{ij} be a number of transitions from state i to state j (i,j= 1,2,…, k). Let the row sum $\sum_{j=1}^{k} m_{ij} = m_{i.}$.

Table 1. Transition Count Matrix

States	**1**	**2**	**.......**	**k**	**Total**
1	m_{11}	m_{12}	....	m_{1k}	$m_{1\bullet}$
2	m_{21}	m_{22}	....	m_{2k}	$m_{2\bullet}$
.	:	:	::::	:	:
.					
k	m_{k1}	m_{k2}	....	m_{kk}	$m_{k\bullet}$
					$m_{\bullet\bullet}$

Let the transition probability matrix of finite Markov Chains be P, then

$$P = \begin{bmatrix} p_{11} & p_{12} \ldots\ldots & p_{1k} \\ p_{21} & p_{22} \ldots\ldots & p_{2k} \\ \vdots & \vdots & \vdots \\ p_{k1} & p_{k2} \ldots\ldots & p_{kk} \end{bmatrix} \quad (1)$$

We are interested with the estimates of the elements $(\hat{P}_{ij})$ of P, . For a given initial state i and a number of trial m_i, the sample of transition counts $(m_{i1} m_{i2} \ldots \ldots m_{ik})$ can be considered as a sample of size m_i from multinomial distribution with probabilities $(p_{i1}\ p_{i2} \ldots \ldots p_{ik})$ such that $\sum_{j=1}^{k} p_{ij} = 1$ (Bhat and Miller, 2002). The probability of this outcome can therefore be given as

$$P(p_{i1}\ p_{i2} \ldots \ldots p_{ik}, m_{i.}) = \frac{m_{i.}}{m_{i1}! m_{i2}! \ldots\ldots m_{ik}!} P_{i1}^{m_{i1}}\ P_{i2}^{m_{i2}} \ldots \ldots P_{ik}^{m_{ik}} \quad (2)$$

As the size of the population tends to infinity the distribution tends to normal. To find the maximum likelihood estimation, we incorporate the condition

$$\sum_{j=1}^{k} p_{ij} = 1 \quad (3)$$

Therefore we have,

$$\hat{p}_{ij} = \frac{m_{ij}}{m_{i.}} \ ; i, j = 1,2, \ldots\ldots, k \quad (4)$$

The inter-state migration data and its transition probability matrix from 2001 is given below.

Table 2: Frequency distribution of Inter-state migrants in 2001.

	Bihar	Delhi	Madhya Pradesh	Maharashtra	Uttar Pradesh
Bihar	0	4,10,275	43,523	2,23,752	2,17,156
Delhi	6,266	0	10,493	35,431	1,45,451

Madhya Pradesh	2,245	42,729	0	2,70,900	1,38,061
Maharashtra	11,834	26,112	1,26,370	0	21,033
Uttar Pradesh	1,06,111	8,60,665	2,78,640	9,01,801	0

Table 3: Frequency distribution of Inter-state migrants in 2001 for males

	Bihar	Delhi	Madhya Pradesh	Maharashtra	Uttar Pradesh
Bihar	0	284300	25458	174483	92109
Delhi	4584	0	4750	18613	60491
Madhya Pradesh	757	22436	0	130135	24933
Maharashtra	2995	12256	3889	0	9361
Uttar Pradesh	15264	477346	99065	626600	0

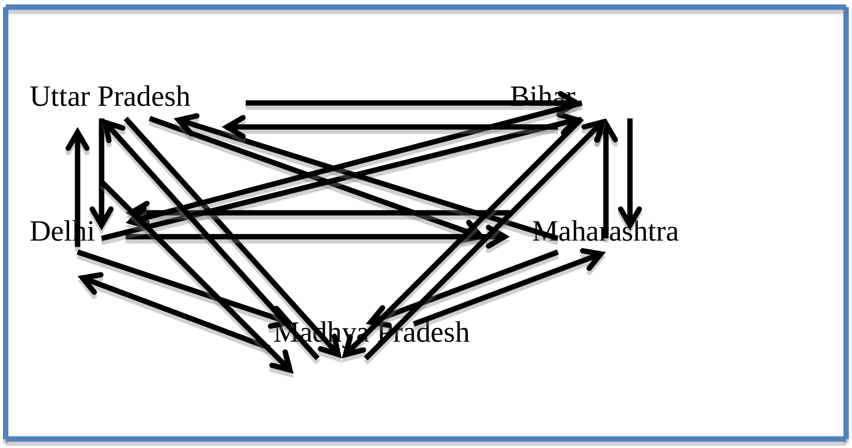
Table 4: Frequency distribution of Inter-state migrants in 2001 for females

	Bihar	Delhi	Madhya Pradesh	Maharashtra	Uttar Pradesh
Bihar	0	125975	18065	49269	125047
Delhi	1682	0	5743	16818	84960
Madhya Pradesh	1488	20293	0	140765	113128
Maharashtra	8839	13856	87481	0	11672
Uttar Pradesh	90849	383319	179575	275201	0

Based on Table-2 and the application of maximum likelihood estimation given by (4), the transition probability matrix for migration in India in 2001 with states S={Bihar, Delhi, Madhya Pradesh, Maharashtra, Uttar Pradesh} is given below:

$$P = \begin{bmatrix} 0.000 & 0.459 & 0.049 & 0.250 & 0.243 \\ 0.032 & 0.000 & 0.053 & 0.179 & 0.736 \\ 0.005 & 0.094 & 0.000 & 0.597 & 0.304 \\ 0.064 & 0.141 & 0.682 & 0.000 & 0.113 \\ 0.049 & 0.401 & 0.130 & 0.420 & 0.000 \end{bmatrix}$$

Figure 1: The graph of transition probability matrix P



To find the mechanism of the migration, first we need to find the transition probability matrix following the procedure given in Bhat and Miller (2002), and Miall (1973) as follow:

Table-5: Transition Count Matrix for k states

States	**1 2 .... k**	**Total**
1 2 . . k	m_{11} m_{12} .... m_{1k} m_{21} m_{22} .... m_{2k} : : :::: : m_{k1} m_{k2} .... m_{kk}	$m_{1\cdot}$ $m_{2\cdot}$: $m_{k\cdot}$
		$m_{\cdot\cdot}$

After that the transition probability matrix is estimated as follows:

$$P^0 = \begin{bmatrix} 0 & \frac{m_{.2}}{m_{..} - m_{.1}} & \frac{m_{.3}}{m_{..} - m_{.1}} \cdots\cdots\cdots & \frac{m_{.k}}{m_{..} - m_{.1}} \\ \frac{m_{.1}}{m_{..} - m_{.2}} & 0 & \frac{m_{.3}}{m_{..} - m_{.2}} \cdots\cdots\cdots\cdots & \frac{m_{.k}}{m_{..} - m_{.2}} \\ . & & . & . \\ . & & . & . \\ . & & . & . \\ \frac{m_{.1}}{m_{..} - m_{.k}} & \frac{m_{.2}}{m_{..} - m_{.2}} & \frac{m_{.3}}{m_{..} - m_{.3}} \cdots\cdots\cdots & 0 \end{bmatrix} \tag{5}$$

Based on (5) and using the data from Table 2 we can obtain the estimates for the entries of transition probabilities matrix.

$$P^0 = \begin{pmatrix} 0.000 & 0.357 & 0.122 & 0.382 & 0.139 \\ 0.050 & 0.000 & 0.181 & 0.564 & 0.205 \\ 0.037 & 0.392 & 0.000 & 0.419 & 0.153 \\ 0.052 & 0.548 & 0.188 & 0.000 & 0.213 \\ 0.038 & 0.399 & 0.137 & 0.427 & 0.000 \end{pmatrix}$$

To check the stability of the transition probability matrix:

$$P^4 = \begin{bmatrix} 0.03927462 & 0.1649941 & 0.2653572 & 0.2571451 & 0.2732289 \\ 0.03715811 & 0.2203041 & 0.2069274 & 0.3407195 & 0.1948909 \\ 0.04526405 & 0.2057565 & 0.3088333 & 0.2450880 & 0.1950581 \\ 0.02845628 & 0.1539134 & 0.1587234 & 0.3535755 & 0.3053314 \\ 0.04163862 & 0.1615480 & 0.2959930 & 0.2304259 & 0.2703946 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.03646371 & 0.1887323 & 0.2214469 & 0.3125130 & 0.2408442 \\ 0.03939296 & 0.1626357 & 0.2710952 & 0.2541284 & 0.2727478 \\ 0.03333820 & 0.1625396 & 0.2055378 & 0.3144334 & 0.2841511 \\ 0.04332827 & 0.2001867 & 0.2902439 & 0.2576666 & 0.2085745 \\ 0.03465995 & 0.1877998 & 0.2027940 & 0.3295788 & 0.2451674 \end{bmatrix}$$

$$P^{15} = \begin{bmatrix} 0.03776822 & 0.1806788 & 0.2418243 & 0.2907712 & 0.2489575 \\ 0.03781610 & 0.1809470 & 0.2421723 & 0.2905979 & 0.2484667 \\ 0.03779394 & 0.1807214 & 0.2420783 & 0.2905638 & 0.2488425 \\ 0.03777608 & 0.1808634 & 0.2417885 & 0.2909012 & 0.2486708 \\ 0.03776276 & 0.1806168 & 0.2418054 & 0.2907555 & 0.2490595 \end{bmatrix}$$

As it is clear from the matrices $P^4, P^5 and\ P^{15}$ that the rows in the matrices are almost identical indicating the stability of transition states after 15th transition or n=15.

For male and female cohort the application of maximum likelihood estimation given by (4), the transition probability matrix for migration in India in 2001 with states S={Bihar, Delhi, Madhya Pradesh, Maharashtra, Uttar Pradesh} is given below:

$$P^m = \begin{bmatrix} 0.000 & 0.493 & 0.044 & 0.303 & 0.160 \\ 0.052 & 0.000 & 0.054 & 0.201 & 0.684 \\ 0.004 & 0.126 & 0.000 & 0.730 & 0.140 \\ 0.047 & 0.193 & 0.612 & 0.000 & 0.147 \\ 0.013 & 0.392 & 0.081 & 0.514 & 0.000 \end{bmatrix}$$

$$P^f = \begin{bmatrix} 0.000 & 0.396 & 0.057 & 0.155 & 0.393 \\ 0.015 & 0.000 & 0.053 & 0.154 & 0.778 \\ 0.005 & 0.074 & 0.000 & 0.511 & 0.410 \\ 0.073 & 0.114 & 0.718 & 0.000 & 0.096 \\ 0.098 & 0.413 & 0.193 & 0.296 & 0.000 \end{bmatrix}$$

Based on (5) and using the data from Table 3 and Table 4 respectively, we can obtain the estimates for the entries of transition probabilities matrix for male and female cohort. The estimated transition probability matrix is given below.

$$P^{m0} = \begin{bmatrix} 0.000 & 0.184 & 0.016 & 0.113 & 0.059 \\ 0.002 & 0.000 & 0.002 & 0.009 & 0.030 \\ 0.000 & 0.012 & 0.000 & 0.067 & 0.013 \\ 0.001 & 0.006 & 0.002 & 0.000 & 0.004 \\ 0.017 & 0.527 & 0.109 & 0.691 & 0.000 \end{bmatrix}$$

$$P^{f0} = \begin{bmatrix} 0.000 & 0.088 & 0.013 & 0.034 & 0.087 \\ 0.001 & 0.000 & 0.003 & 0.010 & 0.052 \\ 0.001 & 0.012 & 0.000 & 0.086 & 0.069 \\ 0.005 & 0.008 & 0.053 & 0.000 & 0.007 \\ 0.055 & 0.233 & 0.109 & 0.167 & 0.000 \end{bmatrix}$$

In order to infer properties of migration mechanism, a matrix of differences of transition probability under the assumption transition probability be (1) and (5). In this case we calculate the difference matrices

$$D = P - P^0 \qquad (6)$$

$$\mathbf{D=} \begin{pmatrix} 0.000 & 0.102 & -0.074 & -0.132 & 0.104 \\ -0.018 & 0.000 & -0.128 & -0.385 & 0.530 \\ -0.032 & -0.298 & 0.000 & 0.178 & 0.152 \\ 0.012 & -0.407 & 0.494 & 0.000 & -0.100 \\ 0.012 & 0.002 & -0.007 & -0.007 & 0.000 \end{pmatrix}$$

Similarly,

The difference matrix for male will be

$$D^m = P - P^{m0}$$

$$D^m = \begin{bmatrix} 0.000 & 0.309 & 0.028 & 0.190 & 0.101 \\ 0.050 & 0.000 & 0.052 & 0.201 & 0.654 \\ 0.004 & 0.114 & 0.000 & 0.663 & 0.127 \\ 0.046 & 0.187 & 0.610 & 0.000 & 0.143 \\ -0.004 & -0.135 & -0.028 & -0.177 & 0.000 \end{bmatrix}$$

The difference matrix for female will be

$$D^f = P - P^{f0}$$

$$D^f = \begin{bmatrix} 0.000 & 0.308 & 0.044 & 0.120 & 0.306 \\ 0.014 & 0.000 & 0.049 & 0.144 & 0.726 \\ 0.004 & 0.061 & 0.000 & 0.425 & 0.342 \\ 0.067 & 0.105 & 0.665 & 0.000 & 0.089 \\ 0.043 & 0.180 & 0.084 & 0.129 & 0.000 \end{bmatrix}$$

Results

The positive elements of the difference matrix D represent those transitions that have higher probability of occurrence than one would have expected from an independent assumption [Bhatt and Miller, 2002]. Therefore, for the data of migration in Indian Census 2001, for whole migrated population, the maximum probability of migration is seen from Delhi to Uttar Pradesh (0.530) among the states considered for study. Also a higher probability of migration is seen from Maharashtra to Madhya Pradesh (0.494). The difference matrix for males and females, D^m *and* D^f respectively shows an increased probability of movement from Delhi to Uttar Pradesh and Maharashtra to Madhya Pradesh as seen for total population. The expected reverse trend of migration from Delhi to UP and Maharashtra to MP can be the result of many factors that needs to be discussed. Return migration can be one of the reasons behind the reverse trend of migration as high amount of migration to metropolitan cities is sweeping the city's infrastructure and environment. Since the states like Delhi, and Maharashtra have outgrown their physical boundaries and increasing migration had forced the near places to develop at faster rate. Urban-agglomerations are becoming equally developed as the developed area e.g. Noida, Ghaziabad, Gurugram etc. The small movement from this states boundaries accounts to major change in migration data.

Further studies can be done for different migration differentials that contribute majorly in change demographic structure of a population. The same analysis can be carried out for genders, age categories and occupation. Thus Markov Chain theory does supply useful insights regarding the observed behavior of a population of migrant cohorts at a given point in time.

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