# Modelling and Forecasting of Age Specific Mortality Rates in India

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**Abstract**

The main objective of this paper is to forecast age specific mortality for India using various stochastic models for the period 2018-2025. The age specific mortality data have been extracted from sample registration system (SRS) during the period 2000-2017. Renshaw and Harberman (RH) model has been identified as best fitted model based on lowest Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) value among all considered models. We have forecasted the age specific mortality rates for India population for the period 2018 to 2025. The forecasted values show a decreasing trend over the periods.

**Key words:** Age specific mortality, Age Period Cohort (APC), Forecasting, RH Model, Lee Carter model.

## Introduction

As of the 2011 census, India's population stood at 1.21 billion, representing a growth rate of 17 Percent over the past decade. Notably, due to substantial demographic shifts, India has surpassed China as the second most populous country in the world. India holds a unique distinction with its favorable demographic dividend, which can serve as a catalyst to boost economic development. Policymakers can examine demographic aspects of mortality and fertility to expedite economic progress. Mortality stands out as a critical demographic variable, and its study, characteristics, and forecasting provide insights into population dynamics and trends (Diaz et al., 2018). Mortality data serve as valuable indicators of a population's health, revealing trends in the age pattern of deaths, cause-specific mortality over time, and offering a snapshot of population growth and current health concerns. The age-specific mortality pattern plays an essential role in shaping public health policies by helping to prioritize interventions. According to SRS-based abridged life tables, India's age-specific mortality rates (ASMRs) have shown a decline from 1970-75 to 2014-18. During the period 2014-18, the life expectancy at birth was 68.2 years for males and 70.7 years for females.

Several authors have highlighted changes in life expectancy and sex differentials in mortality (Singh & Ladusingh, 2016; Subramanian et al., 2006). However, literature on the determinants of age-specific mortality and its forecasting in India remains limited. Phenomena such as population growth and mortality reduction are of great interest due to their significant economic and social implications on a country's development. Therefore, it is imperative to

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establish a reliable forecasted mortality rate to inform planning and guide demographers and policymakers in shaping policies for the betterment of the human population. This paper aims to examine age patterns of mortality in India for both genders and evaluate the performance and adequacy of different stochastic mortality models used in the past to select the most suitable model for forecasting mortality rates. A substantial body of literature has focused on mortality forecasting. Lee and Carter (LC) made a significant breakthrough in 1992 when they developed the first stochastic mortality model based on singular value decomposition and time series analysis. Since then, numerous extensions to this model have been proposed. The first extension, utilizing the two-stage approach and least square estimation, was put forth by Wilmoth (1993) and Lee (2000).

Brouhns et al. (2002) introduced an improved fitting method based on maximum likelihood estimation, offering an alternative approach to incorporate the LC model into a Poisson regression framework. Further extensions to the LC model were presented by Renshaw & Haberman (RH) (2003), which included additional terms in the age-period function. The most widely recognized cohort-based extension of the LC model Is RH model introduced in 2006. To address numerical instability in this method, Haberman and Renshaw (2011) assumed age-independence of the cohort effect to simplify the model. Subsequently, models featuring multiple bilinear age-period components were developed by Hyndman and Ullah (2007) and Hatzopoulos and Haberman (2009), using generalized linear models. Wang et al. (2009) employed principal components analysis. However, some of the extensions of LC models are less popular due to the complex behavior exhibited by higher-order period functions, making forecasting challenging.

One of the most popular variants of the LC model was introduced by Cairns et al. (2006), known as the Cairns-Blake-Dowd (CBD) model. This model leverages the linearity of the logit of one-year death probabilities for older ages. Cairns et al. (2009) expanded the original CBD model by incorporating a combination of a quadratic age term and a cohort effect term. Plat (2009) combined features of the LC model and CBD model to create an advanced model suitable for all age groups, capturing the cohort effect. Conversely, the extension of the Plat model by O'Hare and Li (2012) and Borger et al. (2013) proposed another mortality model suitable for lower age groups. It is worth noting that the LC model has not been applied to Indian mortality data, possibly due to the relatively short history of reasonably reliable age-specific mortality figures. The primary contribution of this work lies in implementing various stochastic models in the Indian context.

The remainder of this paper is organized as follows: Section 2 explores various stochastic mortality models and data sources for this study. Section 3 focuses on fitting these models to mortality incidence rates and, using the best-fit model, forecasting ASMRs for India. The final section contains a discussion and the conclusion of the study.

**Data sources**

Data for this study were retrieved from the Sample Registration System (SRS) from 2000 to 2017. SRS is the only source that provides ASMRs for both genders as well as location of residence (rural and urban) for all major Indian states. The SRS uses a dual sampling

system to collect data on deaths in India. The data is collected from a sample of villages and urban blocks, and all deaths that occur in the selected areas are enumerated. This ensures a fairly accurate representation of the country's demographics. The ASMR is the building-block data for the SRS. It is calculated as the number of deaths in a particular age group divided by the population in that age group. The ASMR is reported for five-year age groups, from 0-4 to 85+. The SRS sampling frame is revised in every ten years based on the latest census results. The revision process considers modifications in the sampling design, wider representation of the population, overcoming the limitations in the existing scheme, and meeting additional requirements. The last replacement of the SRS sample was carried out in 2014.

We have taken the age specific death rate ( $D_{xt}$ ) and the central exposure ( $E_{xt}^{c}$ ) for India by gender. For the analytical purpose age data have been taken in five-year interval from 0-4 to85+ and we denote these age groups as 1, 2, 3…,18 respectively.

**Methods Used**

We have used various stochastic models viz., LC model, RH model, APC model, CBD model, and PLAT model. All these models are implemented using the StMoMo package in R software. A detail review on these stochastic models is given below;

***Lee-Carter (LC) model***

Lee and Carter were the first to use stochastic modeling to estimate life expectancy and age-specific mortality in 1992. The LC model is given as;

$$m_{xt} = e^{\alpha_x + \beta_x k_t + \varepsilon_{xt}} \qquad (1)$$

$$\ln(m_{xt}) = \alpha_x + \beta_x k_t + \varepsilon_{xt} \qquad (2)$$

Where; $m_{xt}$ be the central mortality rate with age group $x$, and the calendar year $t$. $\alpha_x$ represent the average shape of age profile which is independent of time effects and $\beta_x$ represent the pattern of deviations from this age profile when the parameter $k_t$ varies according to time and $\varepsilon_{xt}$ are random error which are assumed to be independent and normally distributed with mean 0 and variance $\sigma^2$ i.e., $\varepsilon_{xt} \sim \mathrm{N}(0, \sigma^2)$.

LC model is a combination of two step inference procedures. In the first step, we estimate the parameters by Singular Value Decomposition (SVD) method because this model cannot be fitted by simple regression methods, due to absence of regressor term (Lee and Carter 1992). For identifiability problem of this model Lee-Carter (1992) suggested two constraints which is used to stabilize the model i.e., $\sum_x \beta_x = 1$ and $\sum_t k_t = 0$. Under these assumptions $\alpha_x$ simply considered as the average values over time of $\ln(m_{xt})$. The second step consists of re-estimation of parameter $k_t$ using time-series process.

***Poisson Lee-Carter model***

Brouhns et al. (2002) proposed an alternate technique for fitting the LC model by assuming a Poisson error term setting for the number of deaths w.r.t. the force of mortality. When examining older ages, the advantages of this approach become evident since the logarithm of observed force of mortality at later ages is considerably more variable. Because the absolute number of fatalities at later ages is lower, the Gaussian assumption (as used in the original LC model) is impractical (Brouhns et al 2002). The LC model re-expressed as:

$$D_{xt} \sim \text{Poisson}\left(E_{xt}^{c} m_{xt}\right)$$

Thus, the predictor structure with non-parametric age period term (N=1) and no cohort effect can be defined as:

$$\ln\left(m_{xt}\right) = \alpha_x + \beta_x^{(1)} k_t^{(1)} + \varepsilon_{xt} \qquad (3)$$

We can estimate the parameters by using a maximum likelihood approach.

***Renshaw and Harberman (RH) model***

In the case of the LC model, the model performs poorly for some nations where a cohort effect has been detected in prior historical data. To overcome this problem Renshaw and Harberman (2006) introduce a cohort effect $\gamma_{t-x}$ to capture effects that could be attributed to the year of birth c=t-x in the LC model as follows;

$$\ln\left(m_{xt}\right) = \alpha_x + \beta_x^{(1)} k_t^{(1)} + \beta_x^{(0)} \gamma_{t-x} \qquad (4)$$

The estimation of the parameters of RH model can be done by assuming a Poisson distribution of death counts. Renshaw and Harberman (2006) postulated that cohort effect $\gamma_{t-x}$ is modeled as an Auto Regressive Integrated Moving Average (ARIMA) (1,1,0) process that is independent of $k_t^{(1)}$. For identifiability problem to ensure the model RH (2006) suggested the following parameter constraints $\sum_x \beta_x^{(1)} = 1$, $\sum_t k_t^{(1)} = 0$, $\sum_x \beta_x^{(0)} = 1$ and $\sum_{c=t_1-x_k}^{t_n-x_1} \gamma_c = 0$

The RH model outperforms the previous mentioned model owing to the inclusion of a cohort impact for nations, but it lacks robustness due to the occurrence of several local maxima in the likelihood function. Currie (2006) fitted this model to UK population and observed that the parameters showed some qualitative differences and also the model incorporates the cohort effects.

Due to numerical instability of the above model Renshaw and Harberman again in 2011 propose another model by assuming the age independence of the cohort effect $\left(\beta_x^{(0)} = 1\right)$ to above given model. This is another simple and popular mortality forecasting model. The modified RH model is as follows;

$$\ln\left(m_{xt}\right) = \alpha_x + \beta_x^{(1)} k_t^{(1)} + \gamma_{t-x} \qquad (5)$$

***Currie Age Period Cohort (APC) model***

Currie (2006) proposed a simplified RH (2006) model known as age period cohort (APC) model, where the age, period and cohort effects influence mortality rates independently that is $\beta_x^{(1)} = 1$ and $\beta_x^{(0)} = 1$. This model removed the robustness problem. The APC modeled as follows:

$$\ln\left(m_{xt}\right) = \alpha_x + k_t^{(1)} + \gamma_{t-x} \qquad (6)$$

***Cairns-Blake-Dowd (CBD) model***

Cairns et al. (2006) introduced a two-factor mortality model with two age period term (N=2) and no static function ($\alpha_x$) with no cohort effect $\gamma_{t-x}$. The CBD model follow a binomial distribution w.r.t. the one-year death probability ($q_{xt}$) is defined as:

$$\ln\left(q_{xt}\right) = k_t^{(1)} + (x - \bar{x}) k_t^{(2)} \qquad (7)$$

Where the pre-specified age modulating parameters are $\beta_x^{(1)} = 1$ and $\beta_x^{(2)} = x - \bar{x}$.

In order to project mortality, the time indexes $k_t^{(1)}$ and $k_t^{(2)}$ can be forecasted using ARIMA model with bivariate random walk with drift. This model does not have identifiability problem and no parameter

constraints are present in CBD model. There are three extensions of this model by addition of cohort effect and quadratic age effect in this model. The demerits of CBD models are that they ignored the lower age groups. These models designed for higher age groups. When these models fitted to all range groups the fit quality is relatively poor and the predictions are biologically unreasonable.

***Plat model***

Plat (2009) proposes a model that attempts to incorporate the best elements of previous models while excluding the models' drawbacks. Plat (2009) introduced a four-factor model, there are three age period term (N=3) and having cohort effect which is a combination of LC model and CBD model. This model is suitable for all ages. The PLAT model is defined as:

$$\ln(m_{xt}) = \alpha_x + k_t^{(1)} + (x-\bar{x})k_t^{(2)} + (\bar{x}-x)^+ k_t^{(3)} + \gamma_{t-x} \qquad (8)$$

Where;

$\beta_x^{(1)} = 1$, $\beta_x^{(2)} = \bar{x} - x$, $\beta_x^{(3)} = (\bar{x} - x)^+ = \max(0, \bar{x} - x)$and the cohort effect with pre-specified age modulating parameters $\beta_x^{(0)} = 1$.

Plat (2009) assumes a Poisson distribution of death count. In order to project mortality, the time indexes $k_t^{(1)}, k_t^{(2)}, k_t^{(3)}$ and the cohort effect $\gamma_{t-x}$ can be modeled and forecast using suitable ARIMA processes.

***Generalized Age-Period-Cohort (GAPC) stochastic mortality structure***

Several stochastic mortality models have been created in recent years. Many stochastic mortality models, according to Currie (2016), may be described in terms of either a generalized linear model (GLM) or a generalized non-linear model (GNM), both of which belong to the family of generalized age-period-cohort (GAPC) stochastic mortality models. The GAPC stochastic mortality models provide a unique framework that may be used to anticipate mortality in a variety of ways (Villegas et al. 2017). In GAPC framework force of mortality $\mu_{xt}$ is constant over each age and year. So that force of mortality $\mu_{xt}$ and central death rate $m_{xt}$will coincide. It can assume that number of deaths $d_{xt}$ with either as central exposure $E_{xt}^c$ or as initial exposure $E_{xt}^0$ with ages $x_i$ $(i = 1,2,\ldots,k)$ and years $t_j$ $(j = 1,2,\ldots,n)$. The GAPC stochastic mortality model produces the four parts of specifications (Mc Cullagh and Neder1989). The model considers random variable $D_{xt}$ be the number of death count which either Poisson or binomially distributed, depending upon the type of exposure. Let us assume $D_{xt}$be an independent random variable then;

$D_{xt}$~ Poisson $(E_{xt}^c\, m_{xt})$, (x= 1, 2, 3……X, t= 1, 2, 3……. T) (9)

$D_{xt}$~ Binomial $(E_{xt}^0, q_{xt})$, (x=1, 2, 3…….X, t= 1, 2, 3……T) (10)

The model should be preferred for a good choice of link function. Hunt and Blake (2014) proposed the log link function with central exposure in Poisson distribution, and logit link function for initial exposure or binomial distribution. Thus the g function is defined as;

$$g\left\{E\left(\frac{D_{xt}}{E_{xt}}\right)\right\} = \eta_{xt} \qquad (11)$$

According to Haberman and Renshaw (2011), all models should be fitted using the same distributional assumptions for comparison of model validity. Therefore, we assume log link function with central exposure in Poisson distribution of deaths

for all models considered here. The predictor structure $\eta_{xt}$ is included as the systematic component of the model which consists of a series of factors dependent on age(x) 0-5 to 85+, period(t)2000 to 2017 and year of birth (or cohort) c=t-x for the population of India a. Hunt and Blake (2014) derive the predictor $\eta_{xt}$ as;

$$\eta_{xt} = \alpha_k + \sum_{i=1}^{N} \beta_x^{(i)} k_t^{(i)} + \beta_x^{(0)} \gamma_{t-x} \quad (12)$$

Where;

$\alpha_x$ representing age-increasing pattern of mortality of India for both genders, and $k_t^{(i)}$ indicate the time trend which will be used to forecast future mortality using ARIMA model. $\beta_x^{(i)}$ , i=1, 2..., N describing how the particular mortality effects are distributed over ages which are multiplied by period functions, $k_t^{(i)}$. The term $\gamma_{t-x}$ accounts for the cohort effect with $\beta_x^{(0)}$ modulating its effect across ages. To identify the stochastic mortality, model the following set of parametric constraints are required:

$$\theta = \left(\alpha_x, \beta_x^{(1)}, ..., \beta_x^{(N)}, k_t^{(1)}, ..., k_t^{(N)}, \beta_x^{(0)}, \gamma_{t-x}\right) \quad (13)$$

### *MLES for considered Stochastic Mortality Models*

Parameter estimation in GAPC stochastic mortality models may be accomplished by maximizing the likelihood function. Log link may be used to write the probability function for a Poisson distribution of death counts as follows:

$$\ell\left(d_{xt}, \hat{d}_{xt}\right) = \sum_x \sum_t w_{xt} \left\{ d_{xt} \log \hat{d}_{xt} - \hat{d}_{xt} - \log \hat{d}_{xt}! \right\} \quad (14)$$

Where;

$\hat{d}_{xt} = E_{xt} g^{-1}\left(\alpha_x + \beta_x^{(i)} k_t^{(i)} + \beta_x^{(0)} \gamma_{t-x}\right)$ be the expected number of deaths

$g^{-1}$ = Predictive model i.e., inverse of the link function $g$

$\omega_{xt}$= weight matrix containing element {0,1}.

### *Goodness of fit*

The residuals of the fitted model are plotted to determine the degree of fit of mortality models. The scale deviance residuals are defined as;

$$r_{xt} = sign\left(d_{xt} - \hat{d}_{xt}\right)\sqrt{\frac{dev(x,t)}{\hat{\phi}}} \quad (15)$$

Where;

$$\hat{\phi} = \frac{D\left(d_{xt} - \hat{d}_{xt}\right)}{k - v}$$

For Poisson random component;

$$dev(x,t) = 2\left[ d_{xt} \log\left(\frac{d_{xt}}{\hat{d}_{xt}}\right) - \left(d_{xt} - \hat{d}_{xt}\right)\right] \quad (16)$$

The total deviance if the model is $D\left(d_{xt}, \hat{d}_{xt}\right) = \sum_x \sum_t \omega_{xt} dev(x,t)$ and $K = \sum_x \sum_t \omega_{xt}$ is the number of observations in the data and $v$ is the effective number of parameters in the model.

In this study, we use scatter plots of residuals (Haberman and Renshaw, 2011) and heat-maps of residuals to assess the degree of fit for various models. For all of the models evaluated in this work, the best fit model was determined by comparing two essential criteria known as Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC).

$$AIC = 2v - 2\ell \text{ and } BIC = v \log k - 2\ell \quad (17)$$

Generally, a model with minimum value of AIC and BIC is considered as the best fit model.

### *Forecasting*

In the GAPC family in order to project mortality rates generally the time index( $k_t^{(i)}$) and cohort index ($\gamma_{t-x}$) are

forecasted by the help of time series methods which comprises of two methods. First, we assume that the period index follows a multivariate random walk with drift.

$$k_t = \delta + k_{t-1} + \xi_t^k \qquad (18)$$

Where $k_t = \left(k_t^{(1)} \ldots k_t^{(N)}\right)^T$, $\xi_t^k \sim \mathcal{N}(0, \sum)$, $\delta$ is an N-dimentional vector of drift parameters and $\sum$ is the $N \times N$ variance-covariance matrix of the multivariate white noise $\xi_t^k$.

The second alternative is to assume that the individual period indexes, $k_t^{(i)}$, i= 1……, N follow a general univariate ARIMA model. Under this approach, the i-th period index, $k_t^{(i)}$, is assumed to be follow an ARIMA $(p_i, q_i, d_i)$ with drift;

$$\Delta^{d_i} k_t^{(i)} = \delta_0^{(i)} + \phi_1^{(i)} \Delta^{d_i} k_{t-1}^{(i)} + \ldots + \phi_{p_i}^{(i)} \Delta^{d_i} k_{t-p_i}^{(i)} + \xi_t^{(i)} + \delta_1^{(i)} \xi_{t-1}^{(i)} + \ldots + \delta_{q_i}^{(i)} \xi_{t-q_i}^{(i)} \qquad (19)$$

Where Δ is the difference operator, $\delta_0^{(i)}$ is the drift parameter, $\phi_1^{(i)}$,……, $\phi_{p_i}^{(i)}$ are the autoregressive coefficients with $\phi_{p_i}^{(i)} \neq 0$,$\delta_1^{(i)}$,……, $\delta_{q_i}^{(i)}$ are the moving average coefficients with $\delta_{q_i}^{(i)} \neq 0$ and $\xi_t^{(i)}$ is a Gaussian white noise process with variance $\sigma_\xi^{(i)}$. Similarly, for cohort index, $\gamma_{t-x}$ follow a univariate ARIMA process which is independent of period index, $k_t$ (Renshaw and Haberman 2006; Carins et al. 2011; Lovasz 2011). In general, assume that $\gamma_c \equiv \gamma_{t-x}$ follow an ARIMA (p, d, q) with drift, so that;

$$\Delta^d \gamma_c = \delta_0 + \phi_1 \Delta^d \gamma_{t-1} + \ldots + \phi_p \Delta^d \gamma_{c-p} + \varepsilon_c + \delta_1 \varepsilon_{c-1} + \ldots + \delta_q \varepsilon_{c-q} \qquad (20)$$

Where $\epsilon_c$ is a Gaussian white noise process with variance $\sigma_\epsilon$.

**Table 1** List of models and constraints

| Models | Notations | Formula | Constraints |
|---|---|---|---|
| LC (1992) | **M1** | $\eta_{xt} = \alpha_x + \beta_x^{(1)} k_t^{(1)}$ | $\sum_x \beta_x = 1, \sum_t k_t = 0$ |
| RH (2006) | **M2** | $\eta_{xt} = \alpha_x + \beta_x^{(1)} k_t^{(1)} + \beta_x^{(0)} \gamma_{t-x}$ | $\sum_x \beta_x^{(1)} = 1$ ,$\sum_t k_t^{(1)} = 0$ ,$\sum_x \beta_x^{(0)} = 1$ , $\sum_{c=t_1-x_k}^{t_n-x_1} \gamma_c = 0$ |
| Extension of RH (2011) | **M3** | $\eta_{xt} = \alpha_x + \beta_x^{(1)} k_t^{(1)} + \gamma_{t-x}$ | $\sum_x \beta_x^{(0)} = 1$ |
| APC (2006) | **M4** | $\eta_{xt} = \alpha_x + k_t^{(1)} + \gamma_{t-x}$ | $\sum_t k_t^{(1)} = 0$ , $\sum_{c=t_1-x_k}^{t_n-x_1} \gamma_c = 0$, $\sum_{c=t_1-x_k}^{t_n-x_1} c\gamma_c = 0$ |
| CBD (2006) | **M5** | $\eta_{xt} = k_t^{(1)} + (x - \bar{x}) k_t^{(2)}$ | **No constraint** |
| PLAT (2009) | **M6** | $\eta_{xt} = \alpha_x + k_t^{(1)} + (x - \bar{x}) k_t^{(2)} + \gamma_{t-x}$ | $\sum_t k_t^{(i)} = 0$, $\sum_{c=t_1-x_k}^{t_n-x_1} \gamma_c = 0$, $\sum_{c=t_1-x_k}^{t_n-x_1} c\gamma_c = 0$, $\sum_{c=t_1-x_k}^{t_n-x_1} c^2\gamma_c = 0$ |

**Analysis and Results**

To exhibit the improvement mortality in India we have plotted age specific mortality rates with time (Figure 1). For the period 2000-2017, Figure 1 depicts the central mortality rates by age group in India. This graph depicts a strong upward trend in death rates by age group over time. In India, it is noticed that younger age groups have a low mortality rate, with the exception of the age group 0-4 which has a high death rate. In India the infant and child mortality are high due to several reasons like malnutrition, illiteracy, hygienic and sanitation etc. In the upper age groups, such as 70-75, 75-80, 80-85, and 85+, a rising tendency may be noticed in above graphs. It is observed that there is decline in death cases of infant and children in current years due to improvements in medical facilities, various health policies implemented by the Government.

Over the period 2000-2017, we displayed the log ($m_{xt}$) value on the y-axis against the age group on the x-axis in Figure 2. It clearly shows that at lower age group its quite high rather than adult age group. In India's male and female instances, it also exhibits an upward tendency in terms of age groups.

**Figure 1** Three-dimensional trend plot of age specific mortality rate of India for the age group 0-4 to 85+ during the year 2000 to 2017.

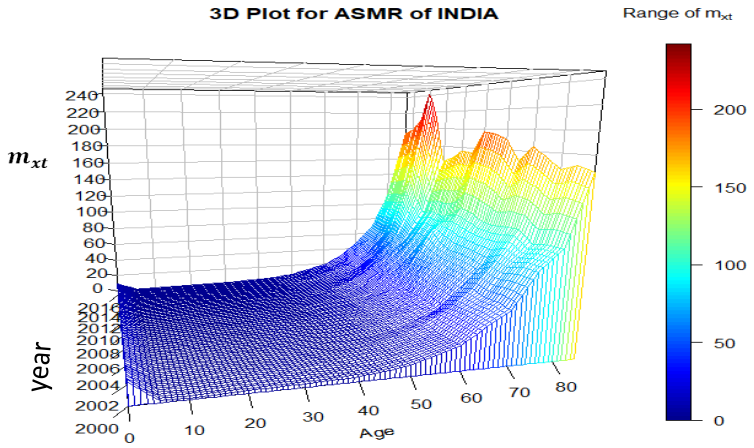
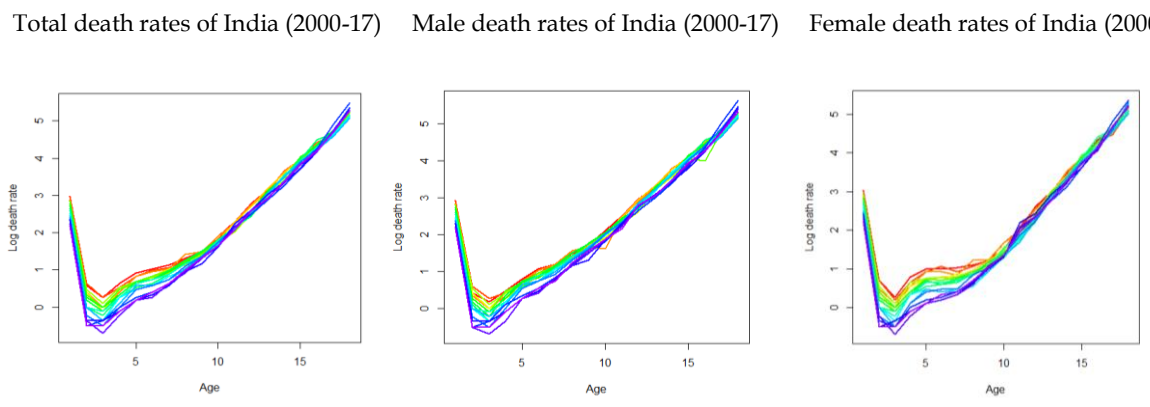



**Figure 2** Log death rate for total, male and female w.r.t. different age groups for the year 2000 to 2017 in India.



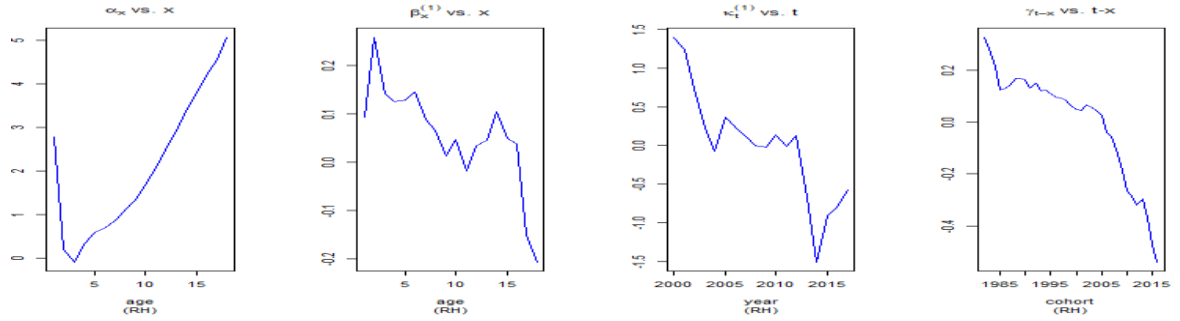
**Parameter estimation**

The different notations viz. static age function $\alpha_x$, age modulating function $\beta_x^{(i)}$, period function $k_t^{(i)}$and cohort age function $\gamma_{t-x}$ are the parameters of all the above considered stochastic mortality models that can be obtained by maximizing the model log-likelihood.

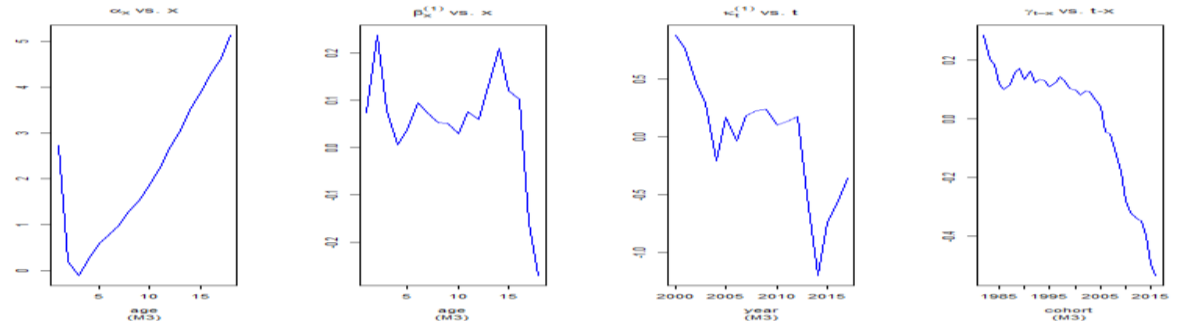
In Figure-3, the estimator $\alpha_x$shows almost linear upward trend in RH model for India. In case of India the estimates for $k_t^{(1)}$ show a decreasing trend for overall and male cases, but in case of females of India it is in increasing pattern. This indicates that in overall general mortality improvement over the period in India. More precisely, cohort estimates of RH model shows decreasing trend over the period.

**Figure 3** Parameter plot for RH model fitted to the India for total, male and female population for different age groups for the period 2000 to 2017.

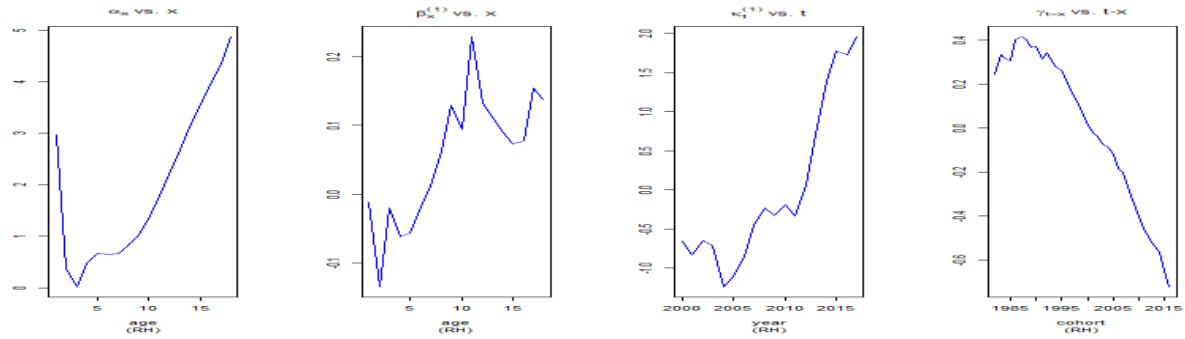
*Parameter plot India Both sexes (RH Model)*




*Parameter plot India Males RH*




*Parameter plot India Females RH*



**Goodness of fit**

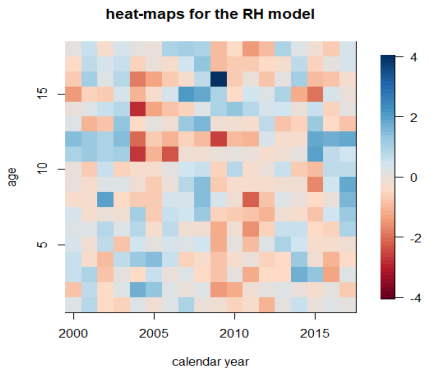
The goodness of fit of these models can be measured by the scaled residual deviance between the observed and fitted data, which depends on the chosen distributional assumptions. In Figure 4 we can see that each line represents the one-year interval. The top row of data represents information about newborns. Each column corresponds to a specific year, while each row represents an age group. In this analysis, we have considered 18 distinct age groups, ranging from 0-4 to 85+, spanning the 18-year period from 2000 to 2017. The colour of each cell is indicative of its deviance residual.

To assess the model, we have grouped these residuals by age and year. In an ideal model, we would expect to see a completely random distribution of colours with no discernible patterns or correlations.

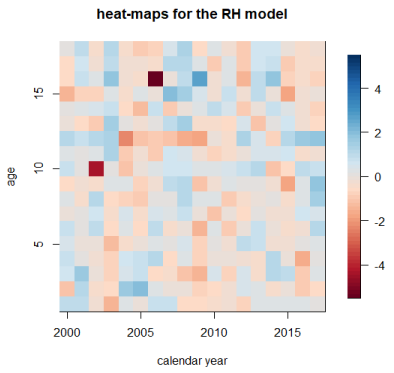
Upon examining the image below, we can observe that, in the case of the RH model, there are no discernible regularities in the colour distribution by year in any instances for India. Similar patterns emerge in the LC, APC, and PLAT models. However, when considering the CBD model, the deviance residuals do not present a completely random pattern.

**Figure 4** Heat-maps of deviance residuals for RH model, fitted to the India population for different age groups for the period 2000 to 2017

Heat Map Plot for RH Model Total India




Heat Map Plot for RH Model Male India




Heat Map Plot for RH Model Female India

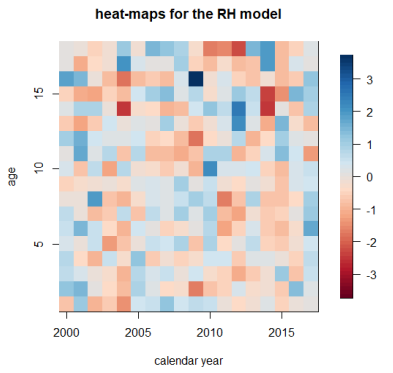
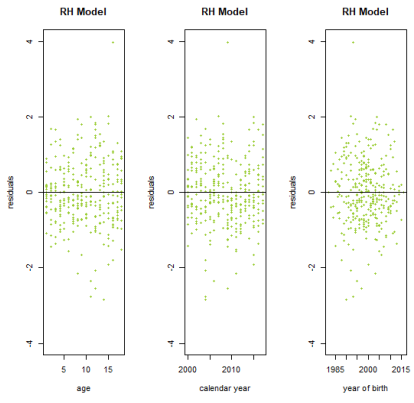
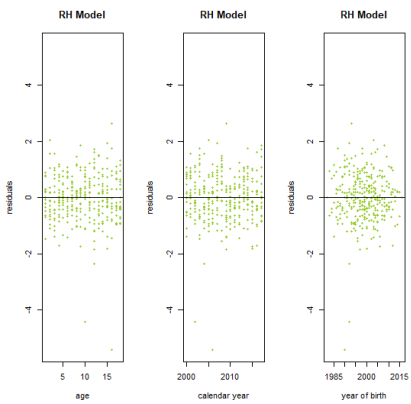



**Figure 5** Scatter plot of deviance residuals of RH model fitted to the Indian total, male and female population for different age groups for the period 2000 to 2017

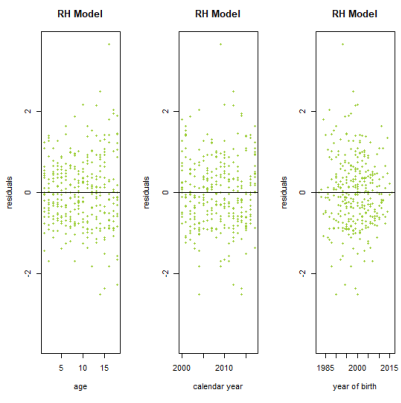
Scatter Plot India Total RH




Scatter Plot India Male RH




Scatter Plot India Female RH



Trends can be identified in specific cases, with certain age groups (0-4, 5-9, and 10-14) appearing predominantly white, blue, or red. This implies that the model's explanatory power for child mortality is relatively weaker compared to adult mortality. Several factors could contribute to this, such as malnutrition, illiteracy, inadequate hygiene, and sanitation practices.

Figure 5 shows the scatter plot of deviance residuals for all fitted model to India for total, male and female population for 0-5 to 85+ age group and the period 2000-2017. We see that each model has age effect, period effect and cohort. Indeed, if the residuals of a model do not exhibit a systematic pattern and instead demonstrate a random distribution, it is considered an ideal or good-fit model.

In the above figure we can see that RH model contains no pattern of residuals and can be considered as good fit models for all cases.

Similar pattern is observed in LC, APC and PLAT models but in case of the model CBD we unable to capture age effect and cohort effect and also it has observed curvature of mortality rate in log scale in all cases of India.

**Information criteria**

Tables 2 and 3 present maximum likelihood estimates along with the corresponding AIC and BIC values for the five selected models fitted for India total, male and female population. It is noted that both criteria lead to the same ranking for RH, LC, PLAT and APC being the best performing models but CBD model hold the worst criteria ranking for the data in all cases. Overall, RH model has been identified as best fit model with lower AIC and BIC value among all considered model in all cases.

**Forecasting**

From the above criteria RH, LC, PLAT and APC are the better fit models. But RH model identified as best fit, so we give special emphasis on forecasting this model with

**Table 2** Model Summary of Indian Total population

| *Model* | Total | | |
|---|---|---|---|
| | Log-likelihood | AIC | BIC |
| LC(M1) | -218474485 | 436949074 | 436949271 |
| **RH (M3)** | **-167990595** | **335981363** | **335981688** |
| APC(M4) | -269735244 | 539470626 | 539470883 |
| CBD(M5) | -59559643284 | 119119286641 | 119119286777 |
| PLAT(M9) | -245758522 | 491517213 | 491517531 |

**Table 3** Model Summary of Male and Female population of India

| *Model* | Male | | | Female | | |
|---|---|---|---|---|---|---|
| | Log-likelihood | AIC | BIC | Log-likelihood | AIC | BIC |
| LC(M1) | -185427352 | 370854809 | 370855006 | -152289873 | 304579851 | 304580048 |
| **RH (M3)** | **-137388098** | **274776369** | **274776694** | **-112595112** | **225190397** | **225190722** |
| APC(M4) | -208719333 | 417438802 | 417439059 | -180475774 | 360951685 | 360951942 |
| CBD(M5) | -29363598779 | 58727197631 | 58727197767 | -32995255921 | 65990511914 | 65990512050 |
| PLAT(M9) | -187266717 | 374533603 | 374533921 | -167622043 | 335244255 | 335244573 |

Table 4 shows ARIMA for the RH model to forecast period and cohort effect applied to India male and female population for all the age group and for the period 2018 to 2025.Here Table 5, shows the forecasted age specific mortality rate of total population of India using best fitted RH model for the years 2018 to 2025. In this table we can see the age specific mortality rate for different years continuously decreases with respect to the calendar years. Comparatively a high mortality rate observed at an early age group (0-4).

We can observe low mortality rate in subsequent ages up to 50-54 age group. The population has lowest mortality rate in the age group 10-15 and highest mortality rate in the age group 85+ throughout all the years.

Almost a constant mortality rate observed among the children (5-9) and adolescent population (10-19) throughout the forecasted years. The forecasted value shows a speed decline in mortality rates among older population with the age group 60-64 to 75-79. These rapid decline in mortality among older persons may be due to the medical advancement and improved health care provided to the older persons. Surprisingly, the mortality rate for the population in the age group 80-85 and 85+ increases with almost more than 100 and 200 respectively.

A similar pattern of mortality rate may be expected in future days for both male and female populations as shown in table 6. Overall, we can observe from the table above that there is a declining tendency of age-specific mortality from 2018 to 2025.

**Table 4** Selected ARIMA (p, d, q) for the model RH with period index and cohort index

| **Model** | $K_t^{(1)}$ | $\gamma_{t-x}$ |
|---|---|---|
| RH | ARIMA (0, 1, 0) with drift | ARIMA (1, 1, 0) with drift |

**Table 5** Forecasted value of age specific mortality rate in India using RH model

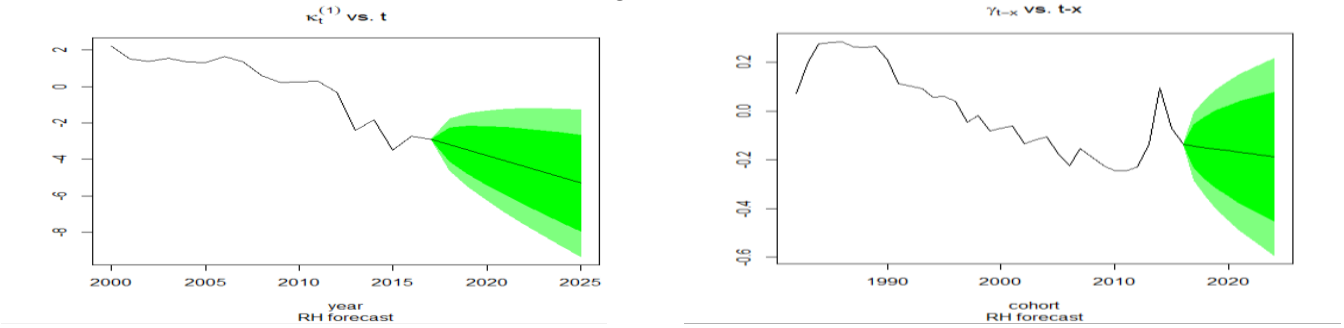
| Age groups | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|---|---|---|
| 0-4 | 8.4 | 8.1 | 7.8 | 7.5 | 7.2 | 6.9 | 6.7 | 6.4 |
| 5-9 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 | 0.4 | 0.4 | 0.4 |
| 10-14 | 0.5 | 0.5 | 0.5 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| 15-19 | 0.9 | 0.8 | 0.7 | 0.7 | 0.6 | 0.6 | 0.6 | 0.6 |
| 20-24 | 1.2 | 1.1 | 1 | 0.9 | 0.9 | 0.8 | 0.8 | 0.8 |
| 25-29 | 1.3 | 1.3 | 1.2 | 1.1 | 1 | 0.9 | 0.9 | 0.8 |
| 30-34 | 1.7 | 1.6 | 1.6 | 1.5 | 1.3 | 1.2 | 1.2 | 1.1 |
| 35-39 | 2.2 | 2.1 | 2.1 | 2.1 | 1.9 | 1.7 | 1.6 | 1.5 |
| 40-44 | 3.1 | 2.9 | 2.8 | 2.7 | 2.8 | 2.6 | 2.3 | 2.2 |
| 45-49 | 4.6 | 4.3 | 4 | 3.9 | 3.7 | 3.8 | 3.5 | 3.1 |
| 50-54 | 7.6 | 7.1 | 6.7 | 6.2 | 6.1 | 5.9 | 6.1 | 5.7 |
| 55-59 | 11.3 | 11.1 | 10.3 | 9.7 | 8.9 | 8.7 | 8.4 | 8.5 |
| 60-64 | 18.7 | 17.3 | 17.1 | 15.8 | 14.8 | 13.6 | 13.3 | 12.7 |
| 65-69 | 28.5 | 27.7 | 25.6 | 25 | 23 | 21.3 | 19.5 | 18.9 |
| 70-74 | 45.7 | 44.7 | 43.8 | 40.6 | 39.9 | 37 | 34.6 | 31.8 |
| 75-79 | 68.8 | 67.9 | 66.6 | 65.3 | 60.7 | 59.7 | 55.4 | 51.8 |
| 80-85 | 111.1 | 115.6 | 116.7 | 117 | 117.2 | 111.3 | 111.9 | 106.2 |
| 85+ | 192.6 | 196.5 | 205.9 | 209.1 | 210.9 | 212.8 | 203.3 | 205.7 |

**Table 6** Forecasted value of age specific mortality rates in India for both males and females using RH models

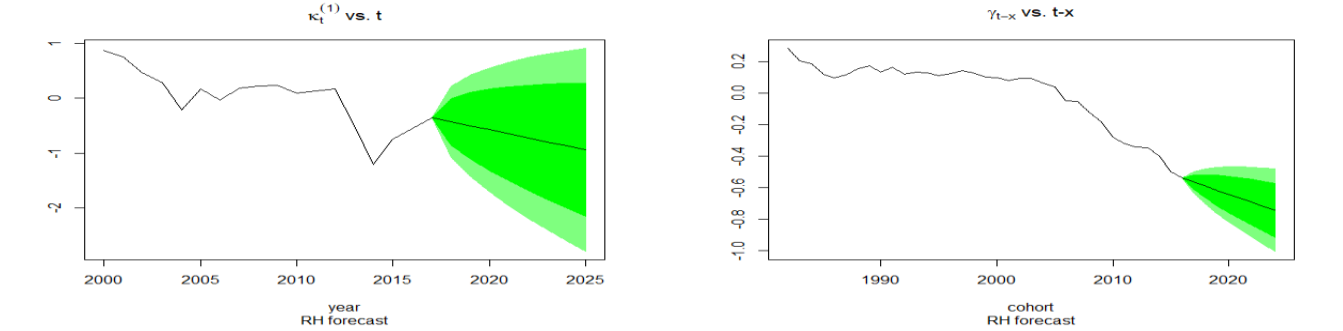
| Age Group | 2018 | | 2019 | | 2020 | | 2021 | | 2022 | | 2023 | | 2024 | | 2025 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Male | Female | Male | Female | Male | Female | Male | Female | Male | Female |
| 0-4 | 8.4 | 8.8 | 8.1 | 8.5 | 7.9 | 8.3 | 7.7 | 8 | 7.4 | 7.8 | 7.2 | 7.6 | 7 | 7.3 | 6.8 | 7.1 |
| 5-9 | 0.6 | 0.5 | 0.6 | 0.5 | 0.6 | 0.5 | 0.6 | 0.4 | 0.5 | 0.4 | 0.5 | 0.4 | 0.5 | 0.4 | 0.5 | 0.4 |
| 10-14 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.4 | 0.5 | 0.4 | 0.5 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| 15-19 | 0.9 | 0.8 | 0.8 | 0.7 | 0.7 | 0.7 | 0.7 | 0.6 | 0.7 | 0.6 | 0.7 | 0.6 | 0.7 | 0.6 | 0.6 | 0.5 |
| 20-24 | 1.3 | 1 | 1.2 | 1 | 1.1 | 0.9 | 1 | 0.8 | 1 | 0.8 | 1 | 0.7 | 0.9 | 0.7 | 0.9 | 0.7 |
| 25-29 | 1.5 | 1.1 | 1.5 | 1.1 | 1.4 | 1 | 1.2 | 1 | 1.2 | 0.9 | 1.1 | 0.8 | 1.1 | 0.8 | 1.1 | 0.8 |
| 30-34 | 1.9 | 1.3 | 1.9 | 1.2 | 1.8 | 1.2 | 1.7 | 1.1 | 1.6 | 1 | 1.5 | 1 | 1.4 | 0.9 | 1.4 | 0.9 |
| 35-39 | 2.7 | 1.7 | 2.6 | 1.6 | 2.5 | 1.6 | 2.5 | 1.6 | 2.4 | 1.5 | 2.1 | 1.4 | 2 | 1.3 | 2 | 1.3 |
| 40-44 | 3.8 | 2.6 | 3.4 | 2.5 | 3.3 | 2.4 | 3.2 | 2.3 | 3.2 | 2.3 | 3 | 2.3 | 2.7 | 2.2 | 2.6 | 2 |
| 45-49 | 5.6 | 3.5 | 5.3 | 3.3 | 4.8 | 3.2 | 4.6 | 3.1 | 4.5 | 3 | 4.5 | 2.9 | 4.2 | 2.9 | 3.8 | 2.7 |
| 50-54 | 8.7 | 7.8 | 8.1 | 7.4 | 7.6 | 7.3 | 6.8 | 7.1 | 6.5 | 7 | 6.4 | 6.9 | 6.3 | 6.9 | 5.9 | 7 |
| 55-59 | 13.4 | 9.9 | 13.2 | 9.9 | 12.3 | 9.3 | 11.5 | 9 | 10.4 | 8.6 | 10 | 8.4 | 9.7 | 8.1 | 9.6 | 8 |
| 60-64 | 21 | 16.4 | 19 | 15.6 | 18.8 | 15.6 | 17.3 | 14.5 | 16.2 | 14 | 14.5 | 13.4 | 13.8 | 12.9 | 13.4 | 12.5 |
| 65-69 | 32.1 | 25.1 | 31 | 24.7 | 28 | 23.5 | 27.4 | 23.4 | 25.1 | 21.7 | 23.4 | 20.8 | 20.9 | 19.9 | 19.7 | 19.1 |
| 70-74 | 50.8 | 38.2 | 48.9 | 38.1 | 47.5 | 37.4 | 43.1 | 35.4 | 42.5 | 35.1 | 39.2 | 32.6 | 36.7 | 31.2 | 33 | 29.7 |
| 75-79 | 74.6 | 58.5 | 73.9 | 57.2 | 71.2 | 57 | 69.3 | 55.9 | 62.9 | 53 | 62.1 | 52.7 | 57.4 | 48.9 | 53.8 | 46.8 |
| 80-85 | 118.2 | 105.3 | 121.2 | 105.9 | 122.4 | 104.7 | 120.2 | 105.6 | 119.2 | 104.9 | 110.3 | 100.7 | 111 | 101.1 | 104.5 | 95 |
| 85+ | 210.9 | 177.2 | 211.5 | 175 | 218.7 | 175.6 | 222.6 | 173.2 | 220.3 | 174.2 | 220.3 | 172.6 | 205.6 | 165.2 | 208.6 | 165.5 |

**Figure 6** Forecast plot of the period index and cohort index of the RH model with random walk with drift $(k_t^{(1)})$ and ARIMA (0,1,0) with drift $(\gamma_{t-x})$ applied to Indian population for the period 2000 to 2017

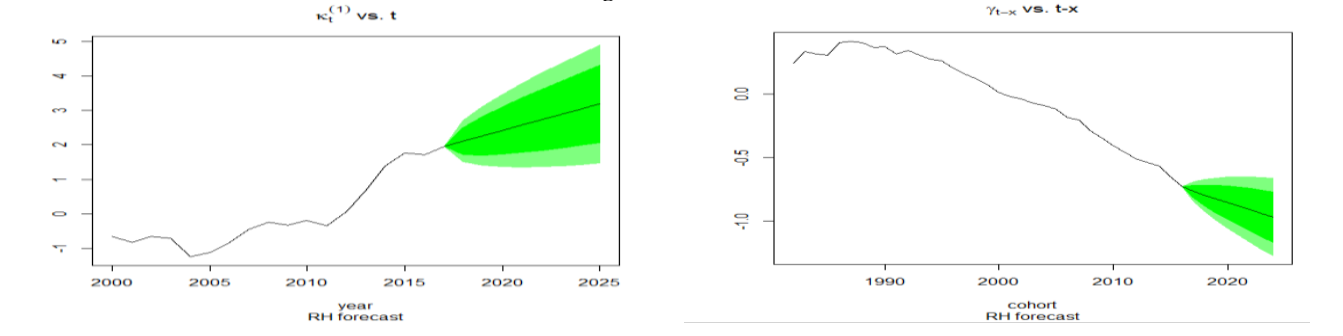
Forecasting Plot for India Total




Forecasting Plot for India Male




Forecasting Plot for India Female



However, for both genders, there was a considerable fluctuation in the 85+ age category of population throughout the forecasted period for India. It was also discovered that the prediction result of male age-specific mortality rate is greater than the female age-specific death rate India.

In Figure 6, the solid black line represents central forecast and shaded region represents 95 percent prediction interval. In both cases we can see the decreasing trend of mortality rate over the period but in case of Indian females it shows an increasing trend, which will have a greater emphasis. The forecast plot shows mortality conditions of Indian male and female population will improve in the future years.

**Discussions and Conclusion**

The death rate in India is compared using stochastic mortality modeling in GAPC framework. We fitted five frequently used stochastic mortality models, LC, RH, APC, CBD, and PLAT, to the 5-year age interval death rate for 18-years (i.e., 2000 to 2017) of the India population. To begin, we plotted the data to see whether there was a pattern, which revealed a decreasing trend throughout the year. We also plotted logarithmic transformation of age specific mortality rate according to. different age groups for the year 2000 to 2017 in India for both the gender. The parameters of GAPC stochastic mortality models were computed using the maximum likelihood estimation approach. We observed that the general pattern of mortality $(\alpha_x)$ for both male and female population of India shown high infant mortality, an accidental hump around ages 20 years and nearly exponential increase in old ages. The sensitivity of $(\beta_x)$ has shown mortality decline at high rate for the age group 25-34 years for female and age groups 15-24 years for male population than other age groups. Mortality index $(k_t)$ has and cohort index $(\gamma_{t-x})$ decreasing trend. To study each model's fitting behavior, specific criteria were considered. The AIC, BIC, and Likelihood values were used to compare the models. The RH model was chosen as the best fit model for India mortality rates based on the selection criteria. In addition, when compared to the CBD model, the LC, APC, and PLAT models have been found as good models. The heat-maps and scattered plot of residuals were also examined. Among other models, the CBD model does not fit well. As a result, we find that the CBD model is ill-suited to the population of India.

In this section, we summarize the above models fitting and forecasting results of this analysis. The result of this findings is compared with the corresponding results obtain from several papers related to India. Our study shows that all models we have taken capture effectively the period effects and cohort effect for both genders except CBD model. Earlier Chavhan & Shinde (2016) have compared ten stochastic mortality models on India population for the age 22-99 years by keeping actuarial application in concern. They observed that the model given by Plat (2009) on the basis of AIC and BIC value is the best model for India male and female mortality.

The effect parameters of model RH, we have forecasted the age specific mortality rates for India population for the period 2018 to 2025. The forecasted value shows a decreasing trend over the periods but exception in females of India with increasing trend.

Overall, we found an improved pattern of mortality in India for both sex in the future. It is a difficult task for the projection models to fit the smaller population data having multiple age-period and cohort terms. The availability of mortality data with extended age range and periods in the future will improve the study of stochastic mortality models. Further, the application of stochastic models for analyzing the mortality rate due to different causes is a possible area of future research.

Forecasting mortality data plays a pivotal role in informing healthcare policy, particularly in the context of India's evolving demographic landscape. By using the model forecasts, policymakers can better identify populations at risk of high mortality, plan for future healthcare needs, and evaluate the effectiveness of health policies. With the country witnessing a gradual shift towards an aging population, the accurate projection of mortality rates and the understanding of age-specific mortality patterns are critical for policymakers. By assessing and forecasting mortality trends, policymakers can strategically allocate resources and develop targeted geriatric care programs, planning social security and healthcare systems.

**Conflict of Interest**

No conflict of interest was reported by all authors.

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