Migration through aspiration: A model based investigation

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Abstract: The study of migration through the different levels of aspiration is very important nowadays because being an immigrant in a world of different aspirations is the reason behind migrating to the spatial world. The reason for the decision to migrate to the spatial world is therefore the different aspiration levels. In this paper, we try to investigate the disparity between two generation's aspiration levels. We investigate the changes in their socioeconomic status which implies their intention towards their desired aspiration. An ambitious person always tries to change his/her socioeconomic status upwards. Hence the probabilities of positional changes explore their level of aspiration. As people migrated to their aspired class at unpredictable intervals of time, so we go for semi-Markov process here. In this paper we try to model the Interval Transition Probabilities according to some optimized assumptions and we will try to propose a measure of association to observe the possible distance between the probabilities of changing aspiration states of two consecutive generation.

Keywords: Aspiration, Semi-Markov Process, Mobility, Occupation, Migration

Introduction

> "Migration is an expression of the human aspiration for dignity, safety and a better future. It is part of the social fabric, part of our very make-up as a human family."
>
> — Ban Ki-moon

"Human migration is an ubiquitous phenomenon that has been shaping countries and societies over centuries of human history" and "that takes place at various scales ranging from individuals to nations". As 'World' is changing toward prosperity, the possible cause of migration that now everyone is looking forward to be a part of this global development potentials and influenced by the worldwide prospect of human life. Everyone is so aspired for this opportunity. However, the coordination between the inner ambience and outer environment of the human being constitutes a different level of aspiration. But along with this aspiration, a competitive narrative lurks in the public spree. The story, however, lies at the heart of all the anecdotes of success and failure. But is this binary reality capable of identifying the fullness of life? In fact, beyond this binary difference there is the satisfaction of human life and the means of living. Since we are concerned with the calculation of differences at different levels of aspiration, these successes and inadequacies are our main focus. Success is a monotonic function of desired aspiration and ability. To achieve success, which is a non-ending process, individual persists to develop qualities and power. And so people are more inclined to migrate to the world. So to say that being an immigrant in a world of different aspirations is the reason behind migrating to the spatial world. The reason for the decision to migrate to the spatial world is therefore the different aspiration levels. No matter how hard the unknown world, the unknown language, the unknown people, the different cultures, the people choose to fulfil their ambitions according to abilities. In some cases,

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people are again compelled to accept “no asking” too. But all those deep misfortunes are not our concern. We will only observe people “coming and going” at different levels of aspiration.

As time has changed, so has the generational “thinking”. Likewise the attitude towards aspiration. We are only interested in finding the magnitude of that change. People can aspire to education, work, socioeconomic power status or standard of livings. The importance of each is substantial and distinct. But we will be specific in this discussion to the different level of occupational aspirations of two respective generations. Since occupational aspiration directly regulates occupational migration, we will be limited to exploring this function for two generations in this discussion.

Let us first try to define aspiration of human population. Aspiration is a complex psycho-logical phenomenon. We can say aspiration is a resultant component of human desire and target for more than that what human has already achieved. Measuring the aspirational levels is somewhat difficult. It may be calculated by some goal discrepancy score, (JohnR. Hills(1955)). Or, as given by (Diecidue et al (2008)), where a simple model being proposed by using the probabilities of reaching to aspiration level, termed as success and other than that considered as failure. Also they combine an aspiration level with expected utility. This will create the behavioural foundation of the given model. Also they include some risk factors to get the closest result. In another paper, (Ray (2002)) discussed some aspect of poverty, which has related closely with a failure of aspirations. He pointed out that the concept of aspirations is related to issues of poverty along with associated ills like poor health etc. He also developed the concept of an aspirations window, which is formed from an individual's cognitive world. He mentioned that everyone draws their aspirations from the lives, achievements, or ideals of those who exist in her aspirations window. As we know that aspiration is a multidimensional factor where, people aspired to get a better standard of living along with dignity, good health, recognition, political power, or the urge to dominate others on religious or ethnic grounds etc. So, socio-economic hierarchy may be a determinant factor for different levels of aspirations which may complement one another, or they may be mutual substitutes. (Carling and Collins (2018)) mentioned the words “Aspiration” and “desire” as the common terms in migration. They visualized that, if migration is a question of human subject then aspiration and desire can be related with three conditions, i.e., a relation to migration possibilities or a potential or other’s relation to mobility. He conceptualize this as migration aspirations. With some other aspects of occupational aspirations which generally lead to occupational migration, we start with some existing works on occupational migration before proceeding our discussion. There exists a large literature on occupational mobility. Occupational mobility based on semi-Markov processes which have been published by Ginsberg (1971, 1972 a,1972 b, 1973, 1978 a, 1978 b). Mehlmann (1979), Bartholomew (1982), McClean (1978,1980, 1986, 1993) and Mukherjee and Chattopadhyay (1989) showed how to use hypothetical grades to reduce certain semi-Markov models to a simple Markov formulation. Bartholomew et al. (1991) observed that any attempt to describe wastage patterns must reckon with the fact that an individual's propensity to leave a job depends on a great many factors, both personal and environmental. Silcock H (1954) summarized the state of statistical knowledge on the matter by giving a list of factors affecting propensity to leave. Tsantas N (1995) has introduced the method for computing the expectations and central quadratic moments of the number of individuals in the various classes at discrete points of time as non-homogeneous Markov system. Tsantas N (2001) has studied the

Asymptotic behaviour of a time dependent Markov model in a stochastic environment, with special relevance to manpower systems. Dimitriou V. A. and Tsantas N (2009) have dealt the exercise of recruitment control to a time dependent, hierarchical system which incorporates training classes as well as two streams of recruitment; one coming from the outside environment and another from an auxiliary external system. Dimitriou V. A.and Tsantas N (2012) have proposed a continuous time semi-Markov hierarchical man-power planning model that incorporates the need of the employees to attend seminars, so as to enhance their prospects, as well as the organization's intention to avoid situations concerning unavailability in skilled personnel when needed. Chattopadhyay and Gupta (2003) have proposed a model of occupational mobility based on Markov process in a half open and half closed system. Chattopadhyay and Khan (2004) have extensively studied the nature of job changes of staff members of an university on the basis of stochastic modelling. Mukherjee and Chattopadhyay (1989), Chattopadhyay and Gupta(2007) and Gupta and Ghoshal (2016) have developed the solution to the problem related to recruitment and promotion of staff members in case of an airline in different situations.

In this paper, we want to connect the tradition of ambition with the tradition of migration. And this attachment may also in some cases be a distraction. Therefore, from the contrary to the conventional ideas, this inquiry should be carried out on the basis of scientific relevance. Therefore, this finding will be based on probability theory. Traditional ambition means that here we are talking about ambitions of two successive generations. And we are interested to find the difference between two generation's aspiration levels from the possible changes in their socioeconomic status as socioeconomic status is the main holder of ambition, we will accept it as the basis. We tried to model the different aspiration level of two generations over time because human life is the period from the beginning to the end of ambition. Therefore, denying the role of time means mistaking to understand this range of highs. In section 2, we discussed about the general structure of interval transition probabilities and make the transgression from Markovian process to semi-Markovian process by considering the random behaviour of time for changing the aspiration states. In section 3, we tried to model the Interval Transition Probabilities according to some optimized assumptions. In section 4 we will propose a measure of association to observe the possible distance between the probabilities of changing inspirational states of two consecutive generation. For virtual validation of our study we have used simulation method, which will be discussed in section 5. As a practical proof of the reality we have presented an example based on real life data in section 6 and discuss the conclusion and findings finally.

General structure of probabilities of the interval transition

As we are interested to investigate the disparity between two generation's aspiration levels, we must investigate the changes in their socioeconomic status which implies their intention towards their desired aspiration. An ambitious person always tries to change his/her socioeconomic status upwards. Hence the probabilities of positional changes explore their level of aspiration. So we have to consider the nature of probabilities to develop a true measure in this case. First we will identify some socioeconomic levels and then assume that the same socioeconomic levels are applicable and available to both generations. Let us denote a particular socioeconomic state by i and consider the states over the range $i = 1, 2, ..., N$. If someone starts at i at the beginning of life, let us also denote the probability of entering in state i by p_i for first generation G_1 (say) and p_i' for next generation G_2 (say). Now if he / she is aspired, he / she will change socioeconomic status. If we assume that this state is j, where $j = 1, 2, ..., N$, then there will be a probability of change from the starting state as p_{ij} for the first generation and p_{ij}' for the next generation. Here we can consider the Markov process to measure this probabilities of changing states. Let us denote two TPMs for two generations as,

$P = ((p_{ij}))$ and $P' = ((p_{ij}'))$ respectively, where,

$$p_{ij} \geq 0 \text{ and } i, j = 1, 2, ..., N$$

$$\sum_{j=1}^{N} p_{ij} = 1 \text{ for } i = 1, 2, ..., N$$

and

$$p_{ij}' \geq 0 \text{ for } i, j = 1, 2, ..., N$$

$$\sum_{j=1}^{N} p_{ij}' = 1 \text{ for } i, j = 1, 2, ..., N$$

When we are measuring highers by the probability of socioeconomic status change, an adequate stochastic model must take into account. People migrated to their aspired class at unpredictable intervals of time. Hence we can think of the semi-Markov process as a process whose occupancy of successive states are governed by the transition probabilities of a Markov process, but where stay in any state is described by a positive values random variables which depend on the state presently occupied and on the state to which the next transition will be made. So, the dynamics of the aspirational migration can be well represented by semi-Markov processes where the states are the different levels of aspirations (Gupta and Ghoshal (2016)). This 'time' is called the holding time. Let us denote τ_{ij} and τ_{ij}', the holding time between changes in the two socioeconomic states of two generations G_1 and G_2 consequently . τ_{ij} and τ_{ij}' must be positive integer-valued random variable, each governed by some probability mass function $h_{ij}(.)$ and $h_{ij}'(.)$, called the holding time mass function for a transition from state i to state j. Thus,

$$P(\tau_{ij} = m) = h_{ij}(m) \ for \ m = 1, 2, ...$$

and

$$P(\tau_{ij}' = m') = h_{ij}'(m') \text{ for } m' = 1, 2, ...$$

In our study we consider the times as random variables of Geometric Distribution which describe the amount of time of failure before the first successful transition from state i to state j. So we can say that,

$$h_{ij}(m)=\begin{cases} a(1-a)^{m-1} & \text{for} \quad m=1,2,... \quad \text{and} \quad 0\le a\le 1 \\ 0 \ \text{Otherwise} \end{cases}$$

and

$$h'_{ij}(m')=\begin{cases} a'(1-a')^{m'-1} & \text{for} \quad m'=1,2,... \quad \text{and} \quad 0\le a'\le 1 \\ 0 \ \text{Otherwise} \end{cases}$$

By assuming $\overline{\tau}_{ij}=\frac{1}{a}$, and $\overline{\tau}'_{ij}=\frac{1}{a'}$ with different parameters, the mean of all holding time distributions for two generations are finite and that all holding times are at least one time unit in length,

$$h_{ij}(0)=0$$

and

$$h'_{ij}(0)=0$$

We will get N^2 holding time mass functions for each generation which describes our discrete time SMP completely. If the time to move from one state i to another state j be τ_{ij} and τ'_{ij}, then after reaching state j it again holds for some time τ_{jk} and τ'_{jk} according to the mass function $h_{ij}(.)$ and $h'_{jk}(.)$ to transit to another state k immediately with probabilities, $p_{j1}, p_{j2}, ..., p_{jN}$ and $p'_{j1}, p'_{j2}, ..., p'_{jN}$, respectively for two generations G_1, G_2 and so on. This can be called the trajectory of the journey and we are interested to find the amount of difference between the journey path of two generations.

Also we can find the waiting time in state i, by τ_i and τ'_i with the waiting time mass function $w_j(.)$ and $w'_i(.)$ for two generations respectively, as follows,

$$w_i(m)=\sum_{j=1}^{N} p_{ij}h_{ij}(m)=P(\tau_i=m) \ \ for \ m=1,2,...$$

and

$$w'_i(m')=\sum_{j=1}^{N} p'_{ij}h'_{ij}(m')=P(\tau'_i=m') \ \text{for} \ m'=1,2,...$$

The mean waiting time at state i be $\overline{\tau}_i$ and $\overline{\tau}'_i$ are related to the holding times $\overline{\tau}_{ij}$ and $\overline{\tau}'_{ij}$ by,

$$\overline{\tau}_i=\sum_{j=i}^{N} p_{ij}\overline{\tau}_{ij}$$

and

$$\overline{\tau}'_i=\sum_{j=i}^{N} p'_{ij}\overline{\tau}'_{ij}$$

From the above equations we will get the complementary cumulative probability distributions for the waiting times as,

$$\bar{w}_i(n) = \sum_{m=n+1}^{\infty} w_i(m) = \sum_{m=n+1}^{\infty}\sum_{j=1}^{N} p_{ij} h_{ij}(m) = \sum_{j=1}^{N} p_{ij}\bar{h}_{ij}(n) = P(\tau_i > n)$$

$$\text{where, } \bar{h}_{ij}(n) = \sum_{m=n+1}^{\infty} h_{ij}(m)$$

And

$$\bar{w}'_i(n') = \sum_{m'=n'+1}^{\infty} w'_i(m') = \sum_{m'=n'+1}^{\infty}\sum_{j=1}^{N} p'_{ij} h'_{ij}(m') = \sum_{j=1}^{N} p'_{ij}\bar{h}'_{ij}(n') = P(\tau'_i > n')$$

$$\text{where, } \bar{h}'_{ij}(n') = \sum_{m'=n'+1}^{\infty} h'_{ij}(m')$$

for two generations G_1 and G_2 respectively.

Modelling the interval transition probabilities

The esoteric statistics in semi-Markov Process are the Interval Transition Probabilities, quantities that correspond to the multi-step transition probabilities for Markov process. In semi-Markov process, we use Interval Transition Probability Matrix (ITPM) instead of Transition Probability Matrix in Markov process (J. Medhi (1994)).

Let us define $\psi_{ij}(n)$ and $\psi'_{ij}(n')$ the probability one will be at state j at time n and n' given that they were in state i at time zero and call this the interval transition probability $i \to j$ in the interval $(0, n)$ and $(0, n')$ respectively for two generations G_1 and G_2.Here we must revisit the fact that member of population has entered the state i instead of being there, considering two cases, as,

- to stay in the same state over the period $(0, n)$ and $(0, n')$, i.e., no change in the socioeconomic state in complete life-time.
- to change aspirational state from i to j it needs to make at least one jump to k state between $(0, m)$ and $(0, m')$ where, $0 < m \le n$ and $0 < m' \le n'$.

Now as given by R.A. Howard (2012), we have,

$$\psi_{ij}(n) = \phi_{ij}\ \bar{w}_i(n) + \sum_{k=1}^{N} p_{ik} \sum_{m=0}^{n} h_{ik}(m)\psi_{kj}(n-m) \quad i,\ j = 1, 2, ..., N; n = 0, 1, 2, ...$$

where

$$\phi_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \ne j \end{cases}$$

and

$$\psi'_{ij}(n') = \phi'_{ij}\ \bar{w}'_i(n') + \sum_{k=1}^{N} p_{ik} \sum_{m'=0}^{n'} h'_{ik}(m')\psi'_{kj}(n'-m') \quad i,\ j = 1, 2, ..., N; n' = 0, 1, 2, ...$$

where

$$\phi'_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

which ensures that the term in which it appears occurs only when $i = j$, and the complementary cumulative waiting time probability $\overline{w}_i(n)$ and $\overline{w}'_i(n')$,

where

$\overline{w}_i(n)$ = The probability that the member of population will leave its starting state i at a time greater than n.

and

$\overline{w}'_i(n')$ = The probability that the member of population will leave its starting state i at a time greater than n'

And the second term in the above model describes the probability of the sequence of events where someone makes its first transition from the starting state i to some state k (maybe i itself) at some time m and m' and then proceeds somehow from state k to state j in the remaining time $(n-m)$ and $(n'-m')$ respectively for G_1 and G_2.

Considering the matrix notation we can write,

$$\Psi(n) = \overline{W}(n) + \sum_{m=0}^{n} \left[P \square \; H(m) \right] \Psi(n-m) \qquad n = 0, 1, 2...$$

and

$$\Psi'(n') = \overline{W}'(n') + \sum_{m'=0}^{n} \left[P' \square \; H'(m') \right] \Psi'(n'-m') \qquad n' = 0, 1, 2...$$

where

$$H(m) = \begin{pmatrix} h_{11}(m) & h_{12}(m) & h_{13}(m) & ... & h_{1N}(m) \\ h_{21}(m) & h_{22}(m) & h_{23}(m) & ... & h_{2N}(m) \\ .... & .... & .... & .... & .... \\ h_{(N-1)1}(m) & h_{(N-1)2}(m) & h_{(N-1)3}(m) & ... & h_{(N-1)\mathrm{N}}(m) \\ h_{N1}(m) & h_{N2}(m) & h_{N3}(m) & ... & h_{NN}(m) \end{pmatrix}$$

and

$$H'(m') = \begin{pmatrix} h'_{11}(m') & h'_{12}(m') & h'_{13}(m') & ... & h'_{1N}(m') \\ h'_{21}(m') & h'_{22}(m') & h'_{23}(m') & ... & h'_{2N}(m') \\ .... & .... & .... & .... & .... \\ h'_{(N-1)1}(m') & h'_{(N-1)2}(m') & h'_{(N-1)3}(m') & ... & h'_{(N-1)\mathrm{N}}(m') \\ h'_{N1}(m') & h'_{N2}(m') & h'_{N3}(m') & ... & h'_{NN}(m') \end{pmatrix}$$

$\Psi(n)$ = The ITPM for SMP in $(0, n)$ for G_1 such that, $\Psi(0) = I$ and

$\Psi'(n')$ = The ITPM for SMP in $(0, n')$ for G_2 such that, $\Psi'(0) = I$

$\square$ = Box notation for congruent matrix multiplication

The diagonal waiting time matrix for G_1, $\overline{W}(n) = \{(\phi_{ij}\overline{w}_i(n))\}$ and for G_2, $\overline{W}'(n') = \{(\phi'_{ij}\overline{w}'_i(n'))\}$

where, $\overline{w}_i(n) = \sum_{m=n+1}^{\infty}$ sum of elements in i^{th} row of $[P \square H(m)]$ and $\overline{w}'_i(n') = \sum_{m'=n'+1}^{\infty}$ sum of elements in i^{th} row of $[P' \square H'(m')]$

Due to the fact that the computation depends only on the $[P \square H(m)]$ and the other one $[P' \square H'(m')]$, so let us denote $C(m) = [P \square H(m)]$ and $C'(m') = [P' \square H(m')]$, the core matrices of semi-Markov process for two consecutive generations. The elements $c_{ij}(m)$ of $C(m)$ and $c'_{ij}(m')$ of $C'(m')$ are the probabilities of the joint event that any member of the population of two generations entered state i at time zero and makes its next transition to state j at time m and m' So we have the waiting time mass function for i^{th} state as,

$$\sum_{j=i}^{N} c_{ij}(m) = \sum_{j=i}^{N} p_{ij}h_{ij}(m) = w_i(m)$$

and

$$\sum_{j=i}^{N} c'_{ij}(m') = \sum_{j=i}^{N} p'_{ij}h'_{ij}(m') = w'_i(m')$$

So we can use,

$$\Psi(n) = \overline{W}(n) + \sum_{m=0}^{n} C(m)\Psi(n-m)$$

and

$$\Psi'(n') = \overline{W}'(n') + \sum_{m'=0}^{n'} C'(m')\Psi(n'-m')$$

as our final model for two consecutive generations.

On some measures of association and $\kappa(d)$

Over time, the era has changed along with people's minds and their thinking and mentality. So if we are aiming to measure that change then we need to find a new measurement. Distance between two populations can be measured by Frobenius Norm (Goluband van Loan (1996)) . Other than this we have Kullback-Leibler divergence (Kullback and Leibler (1951)), Mahalanobis Distance (Mahalanobis (1936)), f-divergences (Csiszár (1963), Morimoto (1963) and Ali Silvey (1966)), Hellinger distance (Nikulin (2001)),Bhattacharyya distance (Bhattacharyya (1943)), Rényi entropy (Rényi (1961)) and soon. Here among these large choices of distance-measures we choose one which can accurately reflect our objective. Let us consider two core matrix for two generations G_1 and G_2 as, $C(m)$ and $C'(m')$, as follows,

$$C(m)=\begin{pmatrix} c_{11}(m) & c_{12}(m) & c_{13}(m) & ... & c_{1N}(m) \\ c_{21}(m) & c_{22}(m) & c_{23}(m) & ... & c_{2N}(m) \\ .... & .... & .... & .... & .... \\ c_{(N-1)1}(m) & c_{(N-1)2}(m) & c_{(N-1)3}(m) & ... & c_{(N-1)\mathrm{N}}(m) \\ c_{N1}(m) & c_{N2}(m) & c_{N3}(m) & ... & c_{NN}(m) \end{pmatrix}$$

and

$$C'(m')=\begin{pmatrix} c'_{11}(m') & c'_{12}(m') & c'_{13}(m') & ... & c'_{1N}(m') \\ c'_{21}(m') & c'_{22}(m') & c'_{23}(m') & ... & c'_{2N}(m') \\ .... & .... & .... & .... & .... \\ c'_{(N-1)1}(m') & c'_{(N-1)2}(m') & c'_{(N-1)3}(m') & ... & c'_{(N-1)\mathrm{N}}(m') \\ c'_{N1}(m') & c'_{N2}(m') & c'_{N3}(m') & ... & c'_{NN}(m') \end{pmatrix}$$

Now $c_{ij}(m)$ and $c'_{ij}(m')$ denotes the probabilities of entering in aspirational state j from i in time m and m' for two generations. For determining the distance we will use,

$$\kappa(d)=\max_{1\le i\le n}\ \max_{1\le j\le n}\left|c_{ij}(m)-c'_{ij}(m')\right|$$

as our distance measure which resulted in,

- if $c_{ij}(m)\to 1$ and $c'_{ij}(m')\to 0 \Rightarrow \kappa(d)\to 1$
- if $c_{ij}(m)\to 0$ and $c'_{ij}(m')\to 1 \Rightarrow \kappa(d)\to 1$
- if $c_{ij}(m)\to 1$ and $c'_{ij}(m')\to 1 \Rightarrow \kappa(d)\to 0$
- if $c_{ij}(m)\to 0$ and $c'_{ij}(m')\to 0 \Rightarrow \kappa(d)\to 0$

Now we can comment that,

- $\kappa(d)\to 1 \Rightarrow$ The generational aspirational gap of probabilities is high. But we cannot say that which generation had higher probabilities of changing aspirational levels.
- $\kappa(d)\to 0 \Rightarrow$ The generational aspirational gap of probabilities is low. So that there is no certain change in probabilities of changing aspirational levels.

Simulation study

To find out the possible distribution of the above measure we have studied the simulated data. Since we are looking for a measure of ambition differences between the two generations, we need separate information for both generations. Therefore, the simulation method must be used for both generations. For the first generation, as firstly we need the transition probability matrix of different aspiration states defined according to socio-economic status, we have generated transition probability matrix of some order $(n\times n)$.

Here we generate the first elements of each Rows form Uniform $(0,1)$ distribution. And then we have taken every next element up to n^{th} by generating values from Uniform

$(0, 1-\text{Sum of all previous values of the specific row})$. Then we have calculated the holding time probability mass functions assuming the values of probability of first successful transition in some time m and so on. Now as mentioned in Section 3, we performed the congruent matrix multiplication operation between the TPM and $H(m)$ and we get the core matrix for the first generation. And now again for the next generation we repeat this process to get the core matrix for the second generation. Now the rest is just to calculate the $\kappa(d)$ to meet our objective. Taking the maximum of the maximum row-values of the absolute difference of core matrices of two generation, we get our first value of $\kappa(d)$. Now this whole process be repeated 100000 times to obtain the simulated distribution of the measure. Comparison of averages of $\kappa(d)$ developed from the differences of core matrices under different choices of m, m', a and a' for holding time matrices are presented in Table 1, Table 2, Table 3, Table 4, Table 6 and Table 7 in Appendices. We can comment that the distribution is almost Normal by plotting but the curvature is little bit skewed from normality, so that we rest this comment unanswered.

Case Study

In a small scale primary study among the faculty members of Ballygunge Science College and Basanti Devi College of Kolkata, we found that they were working in different place sat different times and due to professional ambitions, they have also changed their places of work at different times. It is also true that some of them have been at this university since the beginning of their tenure, still to this day. As a result, the purpose of our present study precisely reflected by this fact. Even as we are concerned about the careerist ambition of the previous generation, that of their father or mother, the information available to us about their previous generation from them also allows us to search. Since our quest is to process different levels of desire, we have organized two levels in each of the two generations - Category 1 and Category 2. Here Category 2 is a higher level of desire than Category 1. As found for the first generation, almost everyone worked in different fields, some were independently employed, and therefore, the social norms and financial norms have been accepted as the basis for this generation's careerist aspiration levels. But for the second generation, this is not so complicated. Since everyone is employed in teaching at college and university, the professor has been taken as a Category 2 and the level of Reader, Assistant Professor and Associate Professor etc. as Category 1. Of course it is in harmony with the previous generation. For both generations, the position acquired at the end of the career or the current position has been considered as the end time-point. And the starting point is set for the previous ten years. For example, we consider herein the case of the second generation from $2010-15$ as $(t-1)$ to $2016-20$ as t, in the other case, from $1990-95$ as $(t-1)$ to $1996-00$ as t. In this interim period, all have changed their activities over a random period of time according to the variations of desires. We have found the transition probability matrices for second and first generations respectively as follows,

$$P = \begin{pmatrix} 0.9 & 0.1 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad P' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

We have estimated the probability of changing this ambitiousness by MLE, which is $\hat{a}_{12} = 0.142857143$ and zero for others for second generation and all $a' = 0$ for the first-generation. And as observed, m and m' are found to be 7years and 0 years respectively. The result is presented in Table 5 in Appendices. From the value of $\kappa(d) = 0.005665278$ we can conclude that the amount of maximum value of absolute generational gap of probabilities of being migrant from one category of aspiration level to another is 0.005665278 for the mentioned data.

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Appendices

Table 1: Comparison Matrix of $\kappa(d)$ developed from the differences of $C(m)$ and $C'(m')$ under different choices of m, m', a and a' for $H(m)$ and $H'(m')$ and for simulated TPM of aspiration levels of order 5×5.

$\overline{\kappa}(d)$		$m=5$			$m=10$			$m=15$		
	a a'	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$m'=5$	0.1	0.04384512	0.0452368	0.05600567	0.04357665	0.05562027	0.05607645	0.0475694	0.05602198	0.0560958
	0.5	0.04513477	0.02091429	0.02663169	0.02470113	0.0262635	0.02672963	0.02014889	0.02663734	0.02670167
	0.9	0.04513477	0.02669079	0.0000599177	0.03303728	0.0262635	0.02672963	0.01949981	0.00006549074	0.0000768
$m'=10$	0.1	0.04362232	0.02478748	0.03311929	0.02589161	0.03265817	0.03315725	0.02568468	0.03310271	0.03315826
	0.5	0.05557603	0.02628964	0.0007966815	0.03264581	0.0006532621	0.0008341445	0.01911164	0.0008211682	0.0008335897
	0.9	0.05598669	0.02670035	0.00007688123	0.03310226	0.0008350533	0.5996728e–9	0.01952804	0.0000260801	0.7690353e–9
$m'=15$	0.1	0.04739253	0.02006194	0.01951338	0.02575753	0.01908486	0.01952392	0.01528025	0.01954381	0.01956371
	0.5	0.05611034	0.02670399	0.00006524233	0.03311273	0.0008220561	0.0000260956	0.01958289	0.00002038468	0.00002604981
	0.9	0.0560553	0.02671555	0.00007701336	0.03308319	0.0008351554	$0.7683148e^{-9}$	0.0195633	0.00002610499	$5.997197e^{-15}$

Table 2: Comparison Matrix of $\kappa(d)$ developed from the differences of $C(m)$ and $C'(m')$ under different choices of m, m', a and a' for $H(m)$ and $H'(m')$ and for simulated TPM of aspiration levels of order 10×10

$\overline{\kappa}(d)$		$m=5$			$m=10$			$m=15$		
	a a'	0.1	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
$m'=5$	0.1	0.04950342	0.05038216	0.06005483	0.04907454	0.05960398	0.0600845	0.05233744	0.06003419	0.06014895
	0.5	0.05027892	0.02360327	0.02860669	0.02791558	0.02818668	0.02865908	0.02272945	0.02865956	0.028653
	0.9	0.06004238	0.02858227	6.780292e-05	0.03545256	0.0008551069	8.250648e-05	0.02092597	7.184027e-05	8.240051e-05
$m'=10$	0.1	0.0489112	0.02792432	0.03543998	0.02924923	0.03506574	0.03544662	0.0288498	0.03551594	0.03547158
	0.5	0.05966493	0.02817795	0.0008550099	0.03506497	0.0007365534	0.0008943169	0.02049368	0.0008794363	0.0008953507
	0.9	0.06001965	0.02863006	8.252915e-05	0.0008934741	0.0008941011	6.765333e-10	0.02096523	2.796637e-05	8.239774e-10
$m'=15$	0.1	0.05217312	0.0226743	0.02092079	0.02879228	0.02048879	0.02095086	0.01724607	0.02094907	0.02091139
	0.5	0.06004817	0.02861546	7.197508e-05	0.03550346	0.0008812095	2.795207e-05	0.02094805	2.29767e-05	2.796187e-05
	0.9	0.06005741	0.02865168	8.249553e-05	0.03552948	0.0008940837	8.246868e-10	0.02094564	2.794623e-05	6.780685e-15

Table 3: Comparison Matrix of $\kappa(d)$ developed from the differences of $C(m)$ and $C'(m')$ under random choices of a and a' and different choices of m and m' for $H(m)$ and $H'(m')$ and for simulated TPM of aspiration levels of order 5×5.

$\bar{\kappa}(d)$		$m=5$	$m=10$	$m=15$
	a a'	Random Choice from U(0,1)	Random Choice from U(0,1)	Random Choice from U(0,1)
$m'=5$	Random Choice from U(0,1)	0.04984421	0.04591961	0.0466127
$m'=10$	Random Choice from U(0,1)	0.0456836	0.02111256	0.01830128
$m'=15$	Random Choice from U(0,1)	0.04674852	0.01825194	0.01237996

Table 4: Comparison Matrix of $\kappa(d)$ developed from the differences of $C(m)$ and $C'(m')$ under random choices of a and a' and different choices of m and m' for $H(m)$ and $H'(m')$ and for simulated TPM of aspiration levels of order 10×10.

$\bar{\kappa}(d)$		$m=5$	$m=10$	$m=15$
	a a'	Random Choice from U(0,1)	Random Choice from U(0,1)	Random Choice from U(0,1)
$m'=5$	Random Choice from U(0,1)	0.05778869	0.05465282	0.0559178
$m'=10$	Random Choice from U(0,1)	0.05445481	0.02554747	0.02260173
$m'=15$	Random Choice from U(0,1)	0.05578824	0.02251433	0.01549028

Table 5: The value of $\kappa(d)$ developed from the difference of $C(m)$ and $C'(m')$ under the estimated value of a and a' and observed value of m and m' for $H(m)$ and $H'(m')$ based on the TPM of aspiration levels from real life data

$\kappa(d)$		$m=$ Interval of changing category
	a a'	Estimated
$m'=$ Interval of changing category	Estimated	0.005665278

Table 6: The value of $\kappa(d)$ developed from the difference of $C(m)$ and $C'(m')$ under equal choice of the estimated value of a and a' and observed value of m and m' for $H(m)$ and $H'(m')$ based on the simulated TPM of aspiration levels of order 5×5.

$\kappa(d)$		$m=$ Interval of changing category
	a a'	Estimated
$m'=$ Interval of changing category	Estimated	0.05189718

Table 7: The value of $\bar{\kappa}(d)$ developed from the difference of $C(m)$ and $C'(m')$ under equal choice of the estimated value of a and a' and observed value of m and m' for $H(m)$ and $H'(m')$ based on the simulated TPM of aspiration levels of order 10×10.

$\kappa(d)$		$m=$ Interval of changing category
	a a'	Estimated
$m'=$ Interval of changing category	Estimated	0.04838045