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Research Article

On Estimation of Fertility Measures: Visualizing the Future Courses through Stochastic Model

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Abstract

The fertility enjoys a key role in the domain of demography as the primary factor responsible for rapid population upturn across the globe. There exist a range of measures to assess the levels and trends within and across the populations. Fertility is affected by socio-cultural and consequential behavioral factors which are effective only through biological manifestations. This induces randomness in the human fertility which makes stochastic modeling relevant to unravel the complicated procedure of human fertility. This study proposes a new approach to estimate the fertility measures ASFR, TFR, GRR, NRR, mean length of generation and growth rate with help of a stochastic model under some pertinent plans defining the fertility regulating and stopping behavior of couples.

Introduction

The demographers, researchers and policy makers across the globe have been stimulated by the convulsive population growth in past few decades. Fertility, as one of the three principal demographic components of population dynamics (Cannan1895), plays a crucial role in studying the past, recent and future trends of population and child bearing behavior. For canvassing the changes in population, demographers and researchers usually make use of various measures of fertility and reproduction like age specific fertility rate (ASFR), total fertility rate (TFR), mean age at childbearing (MAC), net reproduction rate (NRR), etc (Statistics NewZealand2009). In general, these estimated measures premising the present and future trends of fertility dynamics are subject to wide fluctuations within and across the populations. Even if two populations have same value of a fertility measure, they may differ with respect to their growth rates. As an illustration, consider total fertility rate (TFR) which is not affected by age composition of the population and is obtained by summing the age specific fertility rates (ASFR) for the population under consideration. ASFR curve has two attributes, shape of the curve and area under it. The farmer signifies the age pattern of fertility while later represents the inclusive fertility levels (as betoken by TFR). The shape of age specific fertility curve is determined by fertility regulating and limiting behavior i.e. how the women start child bearing, space the births and when they stop reproducing. Therefore, two populations having same levels of TFR but different fertility regulating behaviors will differ with respect to their growth rates.

Human fertility is a biological phenomenon which on one hand is affected by a number of socio-cultural factors and on the other hand by behavioral characteristics of the couple (Oberhofer and Reichsthaler 2004). Though the fertility measures estimated from survey data portray the past and present fertility behavior of the population to good order, projecting the future courses of fertility using them needs an afterthought. This is due to fact that the changing socio-demographic characteristics of women like age at marriage, educational level, employment, economic status and her autonomy have been associated with change in ideal family sizes (Upadhyay and Karasek2012) which in turn influences the preferences regarding the composition of their family. Fertility intentions are considered as potent predictors of fertility, even after controlling the background and life course

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events (Schoen et al. 1999). Also, family planning programs have been helpful for the couples in achieving their fertility desires through birth spacing and limiting behaviors. There have been studies establishing the effect of contraception and family planning services on length of birth intervals (Yeakey et al. 2009) and fertility transitions (Tsui2001). However, all these socio-cultural and behavioral conditioning is affective through the biological manifestations only(Sheps and Menken1973). The policy planners need to comprehend in advance about the behavior of various fertility measures under different possible fertility regulating behavior of couples. There have been two facets of this pursuit. First is collection and analysis of empirical evidences and, second is utilizing stochastic models (Shepsand Menken 1973). Stochastic modelling can help both in analyzing the data and visualizing the future courses of fertility under socio-demographic and behavioral variations.

To visualize the effect of these fertility aspirations which may prevail in future or already existing in society, we make use of some indirect modelling techniques. This study provides a new approach towards evaluating the future courses of fertility under some hypothetical fertility regulating behavior, utilizing two stochastic models due to Singh and Bhattacharya (1970) and Singh and Singh(1978). The models were originally proposed to obtain the probabilities of number of births in duration (0,T) considering the possibility that a conception may or may not be complete. Under consideration of some realistic plans defining fertility stopping behavior of women, we employed above cited model to estimate various measures like ASFR, TFR, GRR, MAC, NRR, mean length of generation and intrinsic growth rate for couples following these stopping behaviors. Stopping rules take care of contraceptive practices and preferences of couple regarding desired size and sex composition of their family.

Methodology

Let $X(T)$ denote the number of conceptions to a female during a time interval $(0,T)$ of length T since marriage. Let the successive conceptions to a woman occur at times $Z_1, Z_2, \ldots\ldots, Z_{n+1}$ and $Y_n = Z_{n+1} - Z_n$ is the time between n^{th} and $n+1^{th}$ conception, which is sum of two parts the rest period following the n^{th} conception and time of resumption of fecundable state after the n^{th} conception and the time of $n+1^{th}$ conception. Clearly, $Y_0 = Z_1$ and this interval does not consists of the rest period. The distribution function of Z_{n+1} as given by Singhand Bhattacharya (1970) is:

$$P[Z_{n+1} \leq T] = \sum_{j=0}^{n} n_j \ \theta^j (1-\theta)^{n-j} [1 - e^{-\lambda(t - jh_2 - \overline{n-j}h_1)} \sum_{m=0}^{n} \frac{[\lambda(t - jh_2 - \overline{n-j}h_1)]^m}{m!}] \qquad (1)$$

and,

$$P[Z_1 \leq T] = \left(1 - e^{-\lambda_0 t}\right); \quad P[Z_0 \leq T] = 1 \qquad (2)$$

The expression of $P[Z_{n+1} \leq T]$ given by equation (1) is for the case when for each conception, the conception rate λ assumes identical values. But, there may be cases when all $\lambda's$ values are different. For such cases $P[Z_{n+1} \leq T]$ can be obtained following Singh and Singh(1978)by equation (3) which is given as:

$$P[Z_{n+1} \leq T] = \sum_{j=0}^{n} n_j \ \theta^j (1-\theta)^{n-j} [\sum_{s=0}^{n} \frac{\prod_{\substack{i=0 \\ i \neq s}}^{n} \lambda_i \ [1 - e^{-\lambda_s (t - jh_2 - \overline{n-j}h_1)}]}{\prod_{\substack{i=0 \\ i \neq s}}^{n} (\lambda_i - \lambda_s)}]; \ \ tjh_2 + \overline{n-j}h_1 \qquad (3)$$

where;

Z_{n+1}= time from marriage to $(n+1)^{th}$ conception.

λ_i = conception rate for $(i+1)^{th}$ conception; $\lambda_i 0, \ t0$.

θ = probability of complete conception i.e. a conception results in a live birth.

and, h_1 and h_2 $(h_1 h_2)$= rest periods associated with incomplete and complete conceptions respectively (which are assumed to be constant as a first approximation).Let, α_n is the probability that the women under consideration will not acquire sterility up to the occurrence of n^{th} conception. Therefore, the probability that the woman will conceive at least $(n+1)$ times in (0,t) is

$$H_{n+1} = \alpha_0\alpha_1\alpha_2 \ldots\ldots \alpha_n [\sum_{j=0}^{n} n_j \;\; \theta^j (1-\theta)^{n-j} [1 - e^{-\lambda(t-jh_2-\overline{n-j}h_1)} \sum_{m=0}^{n} \frac{[\lambda(t-jh_2-\overline{n-j}h_1)]^m}{m!}]] \quad (4)$$

where, $tjh_2 + \overline{n-j}h_1$ and all λ_i are identical and,

$$H_{n+1} = \alpha_0\alpha_1\alpha_2 \ldots\ldots \alpha_n [\sum_{j=0}^{n} n_j \;\; \theta^j (1-\theta)^{n-j} [\sum_{s=0}^{n} \frac{\prod_{\substack{i=0\\ i\neq s}}^{n} \lambda_i \, [1-e^{-\lambda_s(t-jh_2-\overline{n-j}h_1)}]}{\prod_{\substack{i=0\\ i\neq s}}^{n} (\lambda_i-\lambda_s)}]] \quad (5)$$

where, $tjh_2 + \overline{n-j}h_1$ and all λ_i are different.

Let $X(t)$ be number of conceptions in marital duration (0,t), then the probability of exactly n conceptions during (0,t) will be,

$$p_n(t) = P[X(t) = n] = H_n(t) - H_{n+1}(t); \quad n = 0,1,2,\ldots.n'-1 \quad (6)$$

For $n = n'$, $p_n(t) = H_n(t)$ where, n'= maximum number of conceptions a women will have. Also, $H_0(t) = 1$ and $H_1(t) = \alpha_0(1-e^{-\lambda t})$

For illustration of the proposed methodology we anticipated $\lambda_0 = 0.65$, $h_1 = 0.5$ years (4 months of gestation and 2 months of post-partum amenorrhea), $h_2 = 1.25$ years (9 months of gestation and 6 months of post-partum amenorrhea) and $\theta = 0.85$. The α_n values will be determined by the fertility regulating and stopping behavior of couples. There may exist several such stopping rules which define the fertility regulating behavior of couples but, we defined only six hypothetical plans which are given below:

Plan A: 15% of the fecund couple become sterile* following first conception, 50% of the remaining fecund couple become sterile* following second conception, 65% of the remaining fecund couples become sterile* at each conceptions following third conception and 92% of the remaining fecund couples become sterile* at each conceptions following sixth conception i.e. $\lambda_i = \lambda_0 = 0.65, \alpha_0 = 1, \alpha_1 = 0.85, \alpha_2 = 0.50, \alpha_3 = \alpha_4 = \alpha_5 = 0.35$ and $\alpha_i = 0.08 \; \forall \; i = 6,7, \ldots$ Also, the couple have no sex preference.

Plan B: All fecund couples use contraceptive of 70% effectiveness following first conception($\lambda_1 = 0.30\lambda_0, \alpha_0 = 1, \alpha_1 = 1$). 30 % of the remaining fecund couple become sterile * following second conception and remaining 70% fecund couple use contraceptive of 75% effectiveness ($\alpha_2 = 0.70, \lambda_2 = 0.25\lambda_0$). Of the rest fecund couple 50% become sterile* and rest 50% use contraceptive of 85% effectiveness following third conception ($\alpha_3 = 0.5, \lambda_3 = 0.15\lambda_0$). Further, of the remaining fecund couple, 75% become sterile* and 25% use contraceptive of 92% effectiveness following fourth conception ($\alpha_4 = 0.25, \lambda_4 = 0.08\lambda_0$). All, couples become sterile* after fourth conception and there is no sex preference.

Plan C: Proceeding as in case of Plan A, this plan is defined by $\lambda_i = \lambda_0 = 0.65; \;\; \alpha_0 = \alpha_1 = 1, \alpha_2 = 0.80$, and $\alpha_i = 0.50 \;\; \forall \;\; i = 3,4,5, \ldots.$ and there is no sex preference.

Plan D: Proceeding as in case of Plan A, this plan is defined by $\lambda_i = \lambda_0 = 0.65; \;\; \alpha_0 = 1, \alpha_1 = 0.92, \alpha_2 = 0.80, \alpha_3 = 0.75, \alpha_4 = \alpha_5 = \alpha_6 = 0.60$ and $\alpha_i = 0.4 \forall \;\; i = 6,7, \ldots.$ and there is no sex preference.

Plan E: Stop child bearing as soon as one male is born or total number of conceptions is 3, whichever comes early. This implies $\lambda_i = \lambda_0 \forall \; i$ and $\alpha_0 = 1, \alpha_1 = 0.438, \alpha_2 = 0.562$ and $\alpha_i = 0 \forall \;\; i = 3,4, \ldots$

Plan F: Stop child bearing as soon as one male and one female are born or total number of conceptions is 3, whichever comes early. This implies $\lambda_i = \lambda_0 \forall \; i$ and $\alpha_0 = 1, \alpha_1 = 1, \alpha_2 = 0.639$ and $\alpha_i = 0 \forall \;\; i = 3,4, \ldots.$

* or use contraceptive of 100% effectiveness

The calculation of α_n's under plans E and F can be done as illustrated here. Anticipating the widely accepted sex ratio at birth of 106 males per thousand females (given that the conception are complete and result in live birth), the conditional probability of a male birth is $p = 0.515$, the conditional probability of a female birth is $q = 1 - p = 0.485$ and as assumed above, probability that a conception is incomplete $1 - \theta = 0.15$. Let us define the events,
L = a conception remains incomplete(foetal loss).
M = a conception is complete and results in a male birth.
F = a conception is complete and results in a female birth.Therefore,
P[L] = $1 - \theta$ = 0.15, P[M]= p_c = 0.438 and P[F] = q_c = 0.412.
Also, $(1 - \theta) + p_c + q_c = 1$

Since none of the women will be sterile before first conception, therefore, $\alpha_0 = 1$ for each plan. The women abide by plan E will stop conceiving further as soon as one male birth takes place or maximum number of conceptions is 3. Thus, among the women following this plan, all those for whom the first conception was complete and resulted in a male birth will stop further conceptions implying that, $\alpha_1 = p_c = 0.438$. Rest of the females who at first order conception experienced either a foetal loss or a complete conception resulting in a female birth, will proceed to second conception and this implies $\alpha_2 = 0.562$. All women following this plan will stop conceiving after third order conception, hence, $\alpha_i = 0 \forall i = 3, 4, 5\ldots\ldots$. Similarly the α_i values for plan F can be calculated. Moreover, We have considered the maximum marital duration to be 24 years, therefore; t = 1, 2, 3,...24.

Given that our main concern is to estimate the fertility measures ASFR, TFR, GRR, Mean age at child bearing, NRR, mean length of generation and growth rate, equations 4, 5 and 6 can be utilized to to meet the intended objectives in following manner, calculations being done for each of the six plans A, B, C, D, E and F separately:

1. Calculate probabilities of exactly n conceptions i.e $p_n(t) (t = 1, 2, 3, \ldots 24)$ in marital duration $(0, t)$ using equations 4, 5 and 6.
2. Calculate average number of conceptions in interval $(0, t)$ using the values of $p_n(t)$and n.
3. In order to get the average number of births b_t (say) in interval $(0, t)$, multiply the average number of conceptions by θ.
4. Calculate the *duration specific fertility rates* for one year duration as:
   $f_{(t-1),t} = b_t - b_{t-1}$ and $f_{0,1} = b_1$; $(t = 1, 2, 3, \ldots 24)$.
5. Sifting the origin of these duration specific fertility rates from 0 to age a, we get *annual age specific fertility rates*$m_a(t - 1, t)$ for a cohort of women getting married at age a.Next, we advance to calculate the estimates of fertility measures under each of the six plans in following way.

1. **Age specific fertility rate (ASFR)**:
(a) In any population women marry at different ages. One can obtain the age specific fertility rate $m_{a'}(t - 1, t)$ for each cohort of women getting married at specified ages $a' = a,\ a + 1,\ a + 2, \ldots$ following previous steps.
(b) Suppose the proportion of women in a population according to their age at marriage $p_m(a')$ (say) is available. Then, we can calculate the age specific fertility rate $m(t - 1, t)$ for whole population by taking weighted average of the age specific fertility rates for each marriage cohort i.e. $m_{a'}(t - 1, t)$, weights being equal to $p_m(a')$.
(c) For illustration purpose, we have considered $p_m(a')$ from NFHS-3 (2005-06) - India data the proportion of women getting married at ages 15, 16, ...25, 25 $^+$ in the state of Uttar Pradesh and evaluated the ASFRs for plans A, B, C, D, E and F separately (Table 5).
Figure 1 provides ASFR plots for each of six plans. It can be viewed that for plans A, E and F most probable fertility contributing age groups are below 26 years of age. While the women following plans B, C and D will continue child bearing up to their late twenties with faire probabilities.

2. **Total fertility rate (TFR)**: Total fertility rate provides the average number of children that would be born to a woman by the time she ended her childbearing, if she were to pass through all her child bearing ages conforming to given age specific fertility rates. The total fertility rate for plans A, B, C, D, E and F can be obtained by summing over the respective age specific fertility rates $m(t -$

$1, t$)(Table 9). As visible from Table 9, the stopping behaviour of couples is affecting the value of TFR to a great extent. For plan A the TFR is close to replacement levels while for plans C and D it's much higher. For plan E TFR falls below replacement level.

3. **Gross reproduction rate(GRR)**: GRR may be defined as average number of daughters that would be born to a woman during her life time if she passed through her child bearing ages conforming to the a given age specific fertility rate. GRR is like TFR, except that it counts only daughters and literally measures "reproduction" i.e. a woman reproducing herself in next generation by having daughters. Having known the TFR, the gross reproduction rate under each of the considered plans can be obtained by multiplying the respective TFRs with 0.485 (the conditional probability of a female birth given the conceptions result in live birth) (Table 9). It is worthwhile to mention that for all the plans, the estimated values of GRR is more than one except for plan E. i.e. among the couples who choose to stop as soon as a male child is born or maximum number of children born to them is three, whichever comes early, the women will not be able to replace themselves in population.

4. **Mean age at child bearing (MAC)**: MAC is nothing but the mean age of fertility schedule or in other words, a measure of central tendency of the fertility schedule. It is an indicator of the average age of mothers when they give birth to their child if they follow the current fertility schedule during their entire life.

(a) For discrete age distribution, the mean age at child bearing can be calculated by utilizing the expression

$$\bar{m} = \frac{\sum_{\alpha}^{\beta} a.m(a).p(a)da}{\sum_{\alpha}^{\beta} m(a)da}$$

where, m(a) is age specific fertility rate at age a and p(a) is proportion of women surviving to age a. As we are calculating annual ASFRs, a may be taken as $t + 0.5$, the mid point of durations $(t - 1, t)$.
(b) Using the SRS based women's abridged life table (2003-2007) for Uttar Pradesh (Table 6), we interpolated the proportion of women surviving to ages $t + 0.5$ i.e. $p(t + 0.5)$, t=15, 16, 17,....25, 25^{+} and obtained the $\bar{m}$ for each plan A, B, C, D, E and F (Table 8)

5.**Net reproduction rate (NRR)**: GRR, like TFR, assumes that the hypothetical cohort of women from their birth through their reproductive life without experiencing mortality. But, for a more realistic assessment of the reproductive potential of a population taking in to mortality, one needs to calculate NRR. Replacement level fertility is said to be reached when NRR=1.0 i.e. surviving women in the population have exactly enough daughters to replace themselves in population.

(a) Net reproduction rate can be approximated using the expression

$$NRR = p(\bar{m}).GRR$$

where; $p(\bar{m})$ is the proportion of women surviving to mean age at child bearing.

(b) Using the SRS based women's abridged life table (2003-07) for Uttar Pradesh, the $p(\bar{m})$ can be interpolated for each $\bar{m}$ value obtained under the six plans and this facilitates the calculation of corresponding net reproduction rates.

6. **Mean length of generation(G)**: Under anticipation of a stable population, the mean length of generation is given by expression:

$G = \bar{m} - \sigma^2.\frac{log_e GRR}{2.\bar{m}}$ where; σ^2 is the age variance of fertility schedule.

7. **Intrinsic growth rate(r):** For a stable population with discrete age distribution, the intrinsic growth rate can be calculated with help of expression:

$$r = \sqrt[G]{NRR} - 1$$

The value of growth rate for couple populations undergoing the fertility schedules governed by plans A, B, C, D, E and F have been given in Table 9. The percentage intrinsic growth rate ensuing plan A is quite close to zero while, for plans C and D it more than 1.5 whereas, plan E leads to a negative growth rate.

Discussions and conclusions

This study is an illustration that how a stochastic model enables us to visualize the future courses of fertility using minimal empirical information and how a single stochastic model can be utilized and moulded to obtain different aspects of fertility assessment. Though the present study did not propose any new model but it illustrates that how utilizing a model proposed to obtain the number of conceptions in given marital duration, one can estimate various fertility measures under assumption of realistic fertility regulation behaviour of couples. This can help new researchers to be aware about the usefulness of modelling. This study demonstrates that how different aspects of a phenomenon can be studied by changing the assumptions underlying a model formulation, when empirical information corresponding to various possible hypothetical situations in the society is unavailable. In addition, these modelling techniques may also be helpful to policy makers since by changing different assumptions they can have an idea that how variation in the fertility stopping and regulating behaviour of couples is going to shape various fertility measures and they can design their plans accordingly. Table 9 furnishes the evidences that how alterations in fertility regulating and limiting behaviour of couples accompany the variations in estimates of fertility measures. Looking at the male preferred stopping rule plan E, it is ascertained that male preference lower downs TFR as well NRR below the replacement levels leading to negative intrinsic growth rate i.e. for male preferred stopping behaviour with small upper limit over family size, the population will not be able to replace itself.

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Table 1: Average number of conceptions in duration (0, t) after marriage for various plans

	Average no. of conceptions in t years					
t	**Plan A**	**Plan B**	**Plan C**	**Plan D**	**Plan E**	**Plan F**
1	0.166	0.156	0.169	0.167	0.158	0.169
2	0.481	0.425	0.498	0.489	0.435	0.498
3	0.977	0.815	1.035	1.007	0.833	1.031
4	1.349	1.091	1.473	1.423	1.101	1.455
5	1.634	1.301	1.846	1.777	1.290	1.793
6	1.852	1.472	2.163	2.085	1.421	2.054
7	2.018	1.619	2.431	2.357	1.512	2.246
8	2.144	1.749	2.653	2.597	1.574	2.381
9	2.238	1.865	2.837	2.807	1.614	2.474
10	2.308	1.971	2.986	2.991	1.641	2.535
11	2.360	2.068	3.108	3.150	1.657	2.574
12	2.399	2.156	3.205	3.286	1.668	2.599
13	2.427	2.237	3.282	3.403	1.674	2.615
14	2.447	2.311	3.342	3.502	1.678	2.625
15	2.462	2.378	3.389	3.586	1.681	2.630
16	2.472	2.440	3.425	3.657	1.682	2.634
17	2.479	2.497	3.452	3.717	1.683	2.636
18	2.485	2.549	3.472	3.768	1.683	2.637
19	2.488	2.596	3.487	3.810	1.684	2.638
20	2.491	2.640	3.498	3.846	1.684	2.638
21	2.492	2.679	3.506	3.876	1.684	2.639
22	2.493	2.716	3.512	3.900	1.684	2.639
23	2.494	2.749	3.516	3.921	1.684	2.639
24	2.495	2.779	3.519	3.937	1.684	2.639

Table 2: Average number of births in duration (0, t) after marriage for various plans

t	Average number of births in (0,t) years					
	Plan A	Plan B	Plan C	Plan D	Plan E	Plan F
1	0.1412	0.1323	0.1436	0.1423	0.135	0.144
2	0.4087	0.3611	0.4229	0.4153	0.37	0.423
3	0.830	0.6931	0.880	0.8563	0.708	0.877
4	1.1467	0.9269	1.2523	1.2093	0.936	1.236
5	1.3891	1.1056	1.5694	1.510	1.096	1.524
6	1.5745	1.2515	1.8389	1.7723	1.208	1.746
7	1.7157	1.3763	2.066	2.0035	1.286	1.909
8	1.8223	1.4866	2.2552	2.2074	1.338	2.024
9	1.9024	1.5856	2.4111	2.3863	1.372	2.103
10	1.9621	1.6755	2.5385	2.5423	1.395	2.154
11	2.0063	1.7575	2.6415	2.6772	1.409	2.188
12	2.0389	1.8325	2.7241	2.7931	1.418	2.209
13	2.0626	1.9012	2.790	2.8922	1.423	2.223
14	2.080	1.9641	2.841	2.9765	1.427	2.231
15	2.0923	2.0216	2.8808	3.048	1.429	2.236
16	2.1012	2.0743	2.9113	3.1085	1.43	2.239
17	2.1074	2.1224	2.9344	3.160	1.431	2.241
18	2.1118	2.1665	2.9516	3.2026	1.431	2.242
19	2.1149	2.2068	2.9643	3.2388	1.431	2.242
20	2.117	2.2437	2.9736	3.2691	1.431	2.243
21	2.1184	2.2774	2.980	3.2943	1.431	2.243
22	2.1194	2.3082	2.9851	3.3153	1.431	2.243
23	2.120	2.3365	2.9885	3.3326	1.431	2.243
24	2.120	2.3623	2.9909	3.3468	1.432	2.243

Table 3: Duration specific fertility rates for various plans

t	Duration specific fertility rates					
	PlanA	Plan B	Plan C	Plan D	Plan E	Plan F
0-1	0.1412	0.13232	0.14356	0.14228	0.134528	0.14356
1-2	0.2675	0.22877	0.27937	0.27306	0.235033	0.27937
2-3	0.4216	0.33198	0.4572	0.44094	0.338451	0.45382
3-4	0.3164	0.23388	0.37221	0.35303	0.22818	0.360
4-5	0.2424	0.17866	0.31704	0.30087	0.160	0.28789
5-6	0.1854	0.14586	0.26948	0.26213	0.11214	0.22174
6-7	0.1411	0.12488	0.2271	0.23118	0.077364	0.16306
7-8	0.1067	0.110	0.18921	0.20391	0.052226	0.11504
8-9	0.080	0.09905	0.15596	0.17896	0.034495	0.07841
9-10	0.060	0.090	0.12734	0.15593	0.022331	0.05193
10-11	0.0442	0.08202	0.10303	0.13489	0.014201	0.0336
11-12	0.0325	0.075	0.08258	0.11593	0.008891	0.02131
12-13	0.0238	0.06866	0.06551	0.09909	0.005491	0.0133
13-14	0.0173	0.06286	0.05138	0.08433	0.00335	0.00818
14-15	0.0124	0.05755	0.040	0.07152	0.002023	0.00497
15-16	0.0089	0.05266	0.030	0.0605	0.00121	0.00299
16-17	0.0063	0.04818	0.02306	0.05107	0.000718	0.00178
17-18	0.0044	0.04408	0.01723	0.04302	0.000423	0.00105
18-19	0.003	0.04031	0.01272	0.03615	0.000247	0.00062
19-20	0.0021	0.03686	0.00929	0.030	0.000144	0.00036
20-21	0.0014	0.03371	0.00671	0.02528	8.32E-05	0.00021
21-22	0.001	0.03084	0.00481	0.02098	4.80E-05	0.00012
22-23	0.0006	0.02821	0.00341	0.01729	2.75E-05	6.9E-05
23-24	0.0004	0.026	0.0024	0.01414	1.56E-05	4E-05

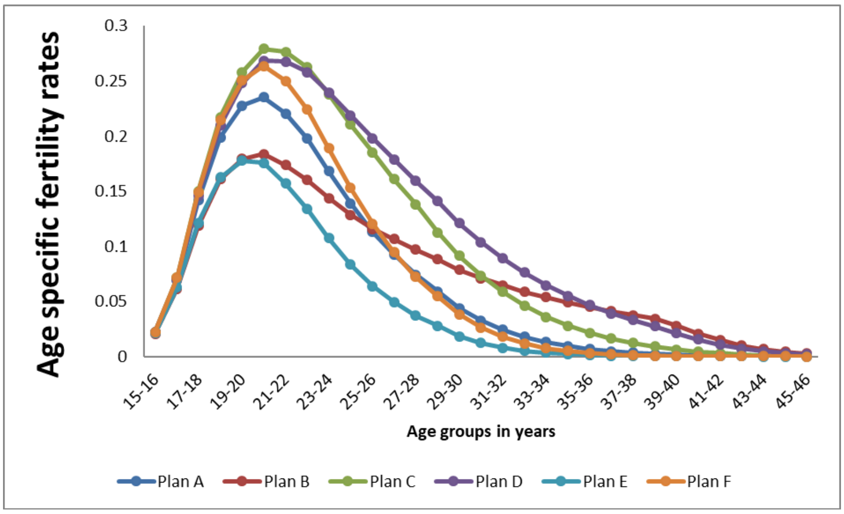

Figure 1: Age specific fertility rate under different plans

Table 4: Proportion of women getting married at different ages in Uttar Pradesh (NFHS-3)

Age at marriage	15	16	17	18	19	20	21	22	23	24	25	25+
Proportion of women $p_m(a')$	0.156	0.194	0.17	0.159	0.1	0.082	0.049	0.029	0.019	0.016	0.01	0.016

Table 5: Age specific fertility rates for various plans

Age group	Plan A	Plan B	Plan C	Plan D	Plan E	Plan F
15-16	0.02202	0.02064	0.0224	0.0222	0.02099	0.0224
16-17	0.06912	0.06136	0.07143	0.0702	0.06276	0.07143
17-18	0.14166	0.11866	0.14993	0.14595	0.12126	0.1494
18-19	0.19907	0.16082	0.21708	0.20966	0.1626	0.21447
19-20	0.22752	0.17929	0.25817	0.24803	0.17757	0.25061
20-21	0.2351	0.18369	0.27922	0.26836	0.17565	0.26345
21-22	0.22052	0.17378	0.27645	0.26765	0.157	0.24991
22-23	0.19751	0.16028	0.26236	0.25802	0.13363	0.22408
23-24	0.16795	0.14372	0.23795	0.23972	0.10715	0.18887
24-25	0.13892	0.12862	0.21044	0.2189	0.08329	0.15297
25-26	0.11313	0.11588	0.1854	0.1979	0.06364	0.12049
26-27	0.09258	0.10639	0.16091	0.1789	0.04937	0.09441
27-28	0.07424	0.09691	0.13792	0.15936	0.03716	0.07211
28-29	0.05903	0.08833	0.11239	0.14088	0.02761	0.05449
29-30	0.04397	0.07863	0.09119	0.121	0.01828	0.03809
30-31	0.03265	0.0709	0.07334	0.1039	0.01208	0.02626
31-32	0.0241	0.06438	0.05841	0.08899	0.0079	0.01774
32-33	0.01767	0.05869	0.04606	0.07599	0.0051	0.01174
33-34	0.01287	0.0536	0.03595	0.06468	0.00325	0.00761
34-35	0.00931	0.049	0.02777	0.05488	0.00204	0.00485
35-36	0.00668	0.04481	0.02125	0.0464	0.00126	0.00304
36-37	0.00476	0.04099	0.01609	0.03911	0.00077	0.00188
37-38	0.00337	0.03749	0.01206	0.03284	0.00047	0.00115
38-39	0.00236	0.0343	0.00896	0.02745	0.00028	0.00069
39-40	0.0016	0.02769	0.00633	0.02105	0.00017	0.00041
40-41	0.00105	0.02075	0.00429	0.01524	9.7E-05	0.00024
41-42	0.00068	0.01497	0.00283	0.01067	5.5E-05	0.00014
42-43	0.00042	0.00994	0.00178	0.00699	3.1E-05	7.8E-05
43-44	0.00026	0.00673	0.00111	0.00463	1.7E-05	4.3E-05
44-45	0.00015	0.00422	0.00066	0.00288	9.2E-06	2.3E-05
45-46	8.8E-05	0.0027	0.00039	0.00181	4.9E-06	1.2E-05

Table 6: Proportion of women surviving to age a, in accordance with SRS abridged life table (2003-07) for Uttar Pradesh

a	**0**	**1**	**5**	**10**	**15**	**20**	**25**
p(a)	1	0.9182	0.9713	0.99094	0.99417	0.99109	0.98772
a	**30**	**35**	**40**	**45**	**50**	**55**	**60**
p(a)	0.98654	0.98314	0.98142	0.98014	0.96161	0.95604	0.93098
a	**65**	**70**	**75**	**80**	**85**		
p(a)	0.88414	0.83359	0.74184	0.66478	0.54839		

Table 7: Interpolated proportion of women surviving to age a

a	**15.5**	**16.5**	**17.5**	**18.5**	**19.5**	**20.5**	**21.5**
p(a)	0.994693	0.9949	0.99426	0.993099	0.991741	0.990504	0.989531
a	**24.5**	**25.5**	**26.5**	**27.5**	**28.5**	**22.5**	**23.5**
p(a)	0.987865	0.9876	0.987409	0.987238	0.987032	0.988793	0.98825
a	**29.5**	**30.5**	**31.5**	**32.5**	**33.5**	**34.5**	**35.5**
p(a)	0.986737	0.986302	0.985721	0.985034	0.984285	0.983517	0.982775
a	**36.5**	**37.5**	**38.5**	**39.5**	**40.5**	**41.5**	**42.5**
p(a)	0.982113	0.9816	0.98129	0.981296	0.981632	0.98216	0.982511
a	**43.5**	**44.5**	**45.5**				
p(a)	0.982308	0.981174	0.978752				

Table 8: Mean age at childbearing $\bar{m}$ and, interpolated proportion of women surviving to age $\bar{m}$ for various stopping rules

Plan	$\bar{m}$	**p($\bar{m}$)**
A	22.25	0.98896
B	25.03	0.987712
C	23.40	0.988296
D	24.53	0.987855
E	21.34	0.989672
F	21.92	0.989197

Table 9: Estimated fertility measures

Plan	**TFR**	**GRR**	**NRR**	**G**	**r**
A	2.120	1.028	1.017	22.23	0.0759
B	2.358	1.144	1.130	24.90	0.4896
C	2.991	1.450	1.433	23.17	1.5538
D	3.344	1.622	1.602	24.16	1.9515
E	1.432	0.694	0.687	21.48	-1.747
F	2.243	1.088	1.076	21.88	0.3354